Scale-dependent background-error covariance modelling, with application to ECMWF's ocean reanalysis system

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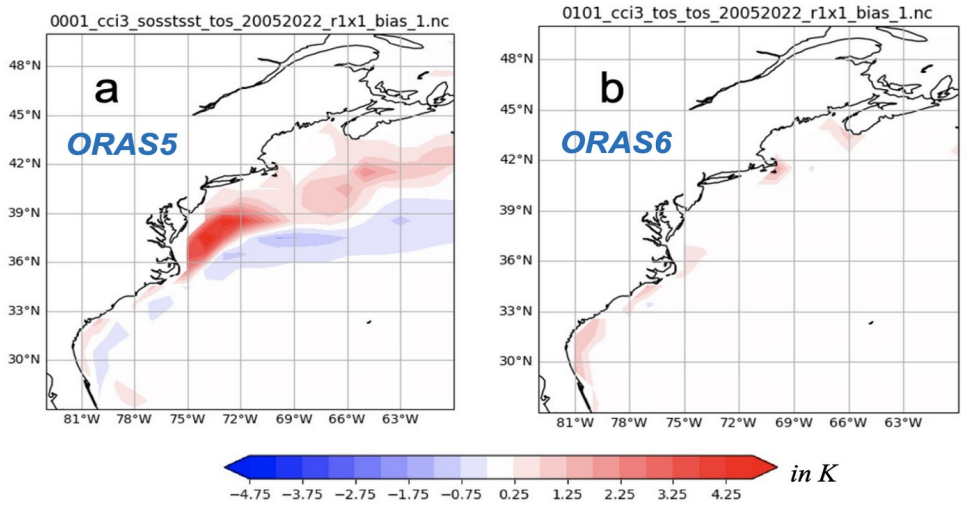
[2]ECMWF, Reading

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Hybrid **B** formulation in ECMWF's ocean reanalysis ORAS6

- Ensemble perturbations (10 members) from an ocean EDA are used to estimate parameters of the background-error covariance model in 3D-Var FGAT (NEMOVAR).
  1) Seasonally-dependent (climatological) estimates of the variances and horizontal correlation length scales.
  2) Flow-dependent estimates of the variances and vertical correlation length scales.
- 1) and 2) are combined in a hybrid formulation of **B**.
- Significant positive impact compared to the strictly parameterised **B** used in ORAS5 (Chrust et al. 2025; Zuo et al. 2026).

SST biases, relative to SST-CCI2,
for 2017-2021



Limitations of the ORAS6 hybrid **B** formulation

- Inability of the diffusion-based correlation model to account for functional relationships that depend on spatial scale.
- The correlation model is characterised by a quasi-Gaussian correlation function with a single, locally-defined diffusion tensor.
- Produces overly localised correlation structures in regions dominated by mesoscale eddies.
- This presentation describes a multi-scale variant of the correlation model.
  1) Incorporates key ideas from scale-dependent localisation (SDL; Buehner and Shlyaeva, 2015).
  2) Can be viewed as an extension of the multi-scale approach of Mirouze et al. (2016), to account for cross-scale correlations and the specification of parameters using ensembles.

Multi-scale **B** formulation

(Weaver and Chrust, submitted to QJRMS)

- For each analysis variable α (where the different α are assumed mutually uncorrelated):

$$\mathbf{B}_{\mathrm{MS}}^{\alpha} = \boldsymbol{\Sigma}\,\boldsymbol{\Gamma}_{\mathrm{g}} \left(\sum_{i=1}^{N_{\mathrm{S}}} \sum_{j=1}^{N_{\mathrm{S}}} \boldsymbol{\Sigma}_i\, \mathbf{C}_{ij}\, \boldsymbol{\Sigma}_j \right) \boldsymbol{\Gamma}_{\mathrm{g}}\, \boldsymbol{\Sigma}$$

$\mathbf{C}_{\mathrm{MS}}$

Global std deviations

Global normalisation factors

Scale-dependent correlation matrix

Scale-dependent std deviations

- The **scale-dependent correlation matrix** is modelled as

$$\mathbf{C}_{ij} = \mathbf{U}_i \mathbf{U}_j^{\mathrm{T}}$$

where $\mathbf{U}_i$ ($\mathbf{U}_j^{\mathrm{T}}$) is a (normalised) diffusion operator whose parameters characterise correlations for scale *i* (scale *j*).

Univariate, scale-dependent localised **B**

(Buehner and Shlyaeva, 2015)

- For each analysis variable α

$$\mathbf{B}^{\alpha}_{\mathrm{SDL}} = \widetilde{\boldsymbol{\Sigma}}\,\widehat{\boldsymbol{\Gamma}}_{\mathrm{g}}\underbrace{\left(\sum_{i=1}^{N_s}\sum_{j=1}^{N_s}\sum_{p=1}^{N_e}\boldsymbol{\Lambda}_{p,i}\mathbf{L}_{ij}\boldsymbol{\Lambda}_{p,j}\right)}_{\mathbf{C}_{\mathrm{SDL}}}\widehat{\boldsymbol{\Gamma}}_{\mathrm{g}}\,\widetilde{\boldsymbol{\Sigma}}$$

Global std deviations

Global normalisation factors

Scale-dependent localisation matrix

Scale-dependent ensemble perturbations

- The **scale-dependent localisation matrix** is modelled as

$$\mathbf{L}_{ij} = \mathbf{U}_i\mathbf{U}_j^{\mathrm{T}}$$

where $\mathbf{U}_i$ ($\mathbf{U}_j^{\mathrm{T}}$) is a (normalised) diffusion operator whose parameters characterise localisation scale *i* (scale *j*).

Separating scales in the ensemble perturbations

- Define $\mathbf{X} = \left(\epsilon'_1 \ \ldots \ \epsilon'_{N_e} \right)$ where $\epsilon'_p = \mathbf{K}_b^{-1} \left\{ \mathbf{x}_p - \frac{1}{N_e} \sum_{l=1}^{N_e} \mathbf{x}_l \right\}$

Inverse of the balance operator

- Separate **X** into scale-dependent ensembles using a spatial (diffusion) filter **F**.

- Example with two scales:

$$\mathbf{X}_1 = \mathbf{F}_1(\mathbf{X})$$ Large-scale ensembles

$$\mathbf{X}_2 = \mathbf{X} - \mathbf{F}_1(\mathbf{X})$$ Smaller-scale ensembles

where $\mathbf{X} = \mathbf{X}_1 + \mathbf{X}_2$

Scale-dependent filtering using a diffusion operator

Effective spectral filtering coefficients

(Weaver and Mirouze, 2013)

Filtering length scale

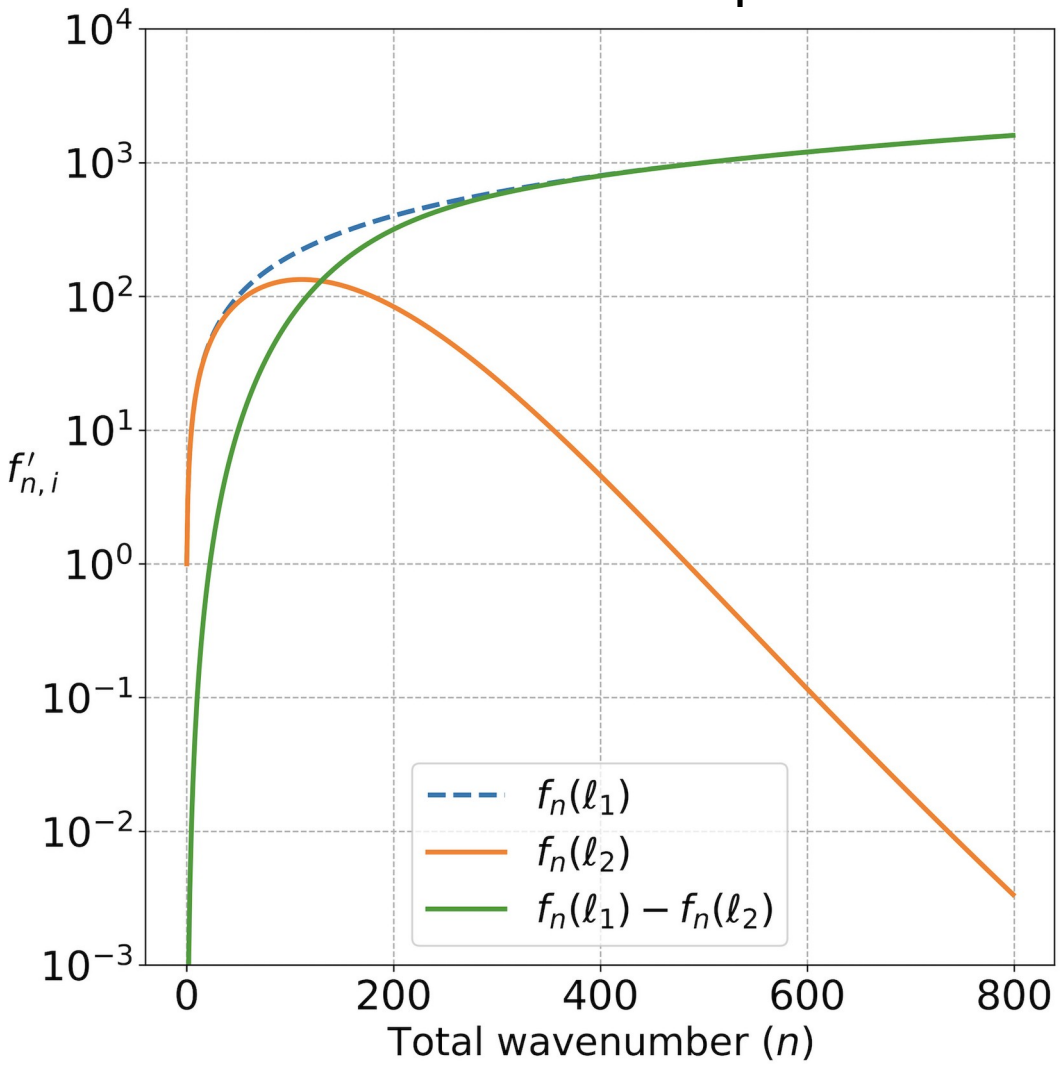
$$f_n = \sqrt{2n+1}\left(1 + \frac{\ell^2}{8a^2(M-1)} n(n+1)\right)^{-M}$$

Earth's radius

Number of implicit diffusion iterations

Here, $\ell_1 = 0$, $\ell_2 = 111$ km and $M = 10$

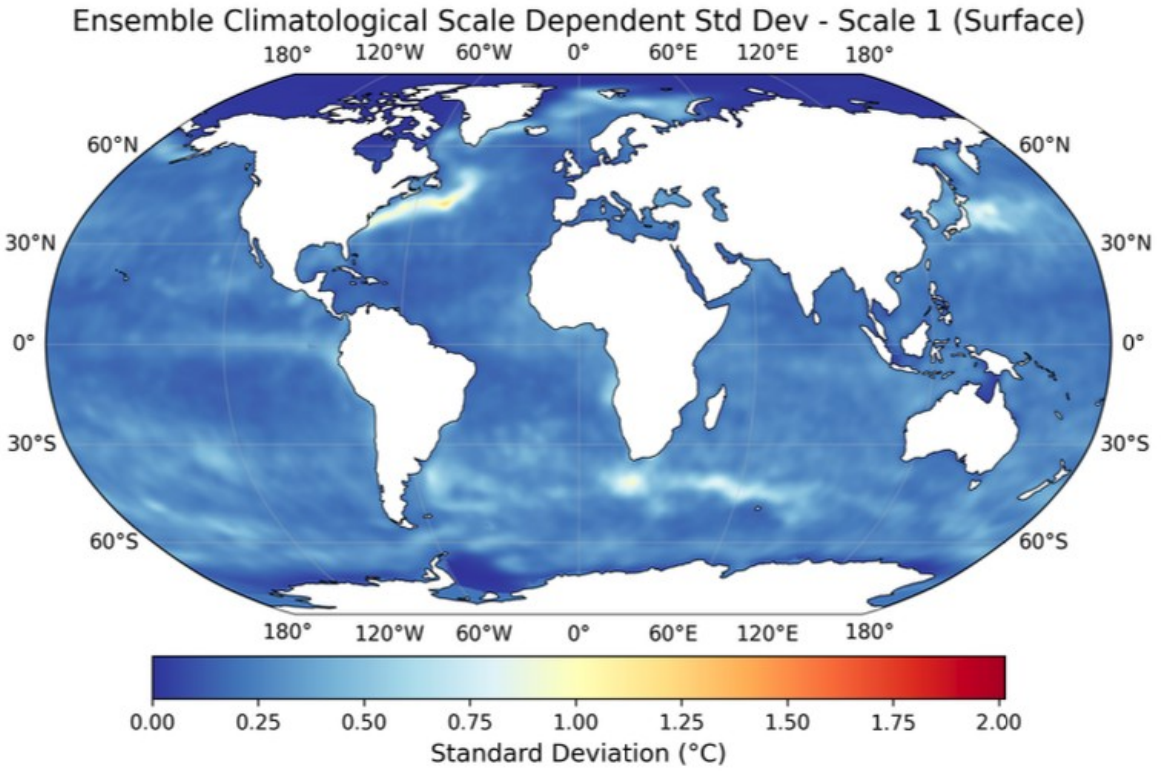
Two-scale example



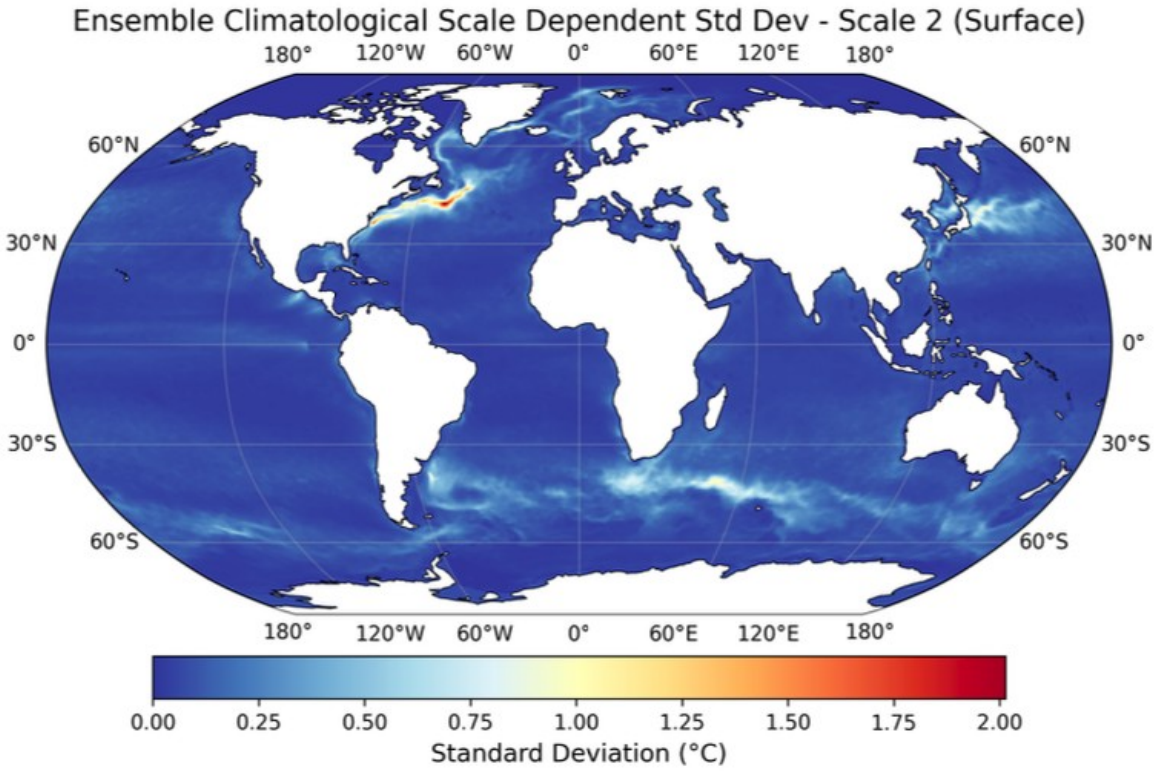
Scale-dependent climatological (DJF) standard deviations for temperature

Filtering length scale = β x local horizontal grid resolution, with β = 3

Large-scale term



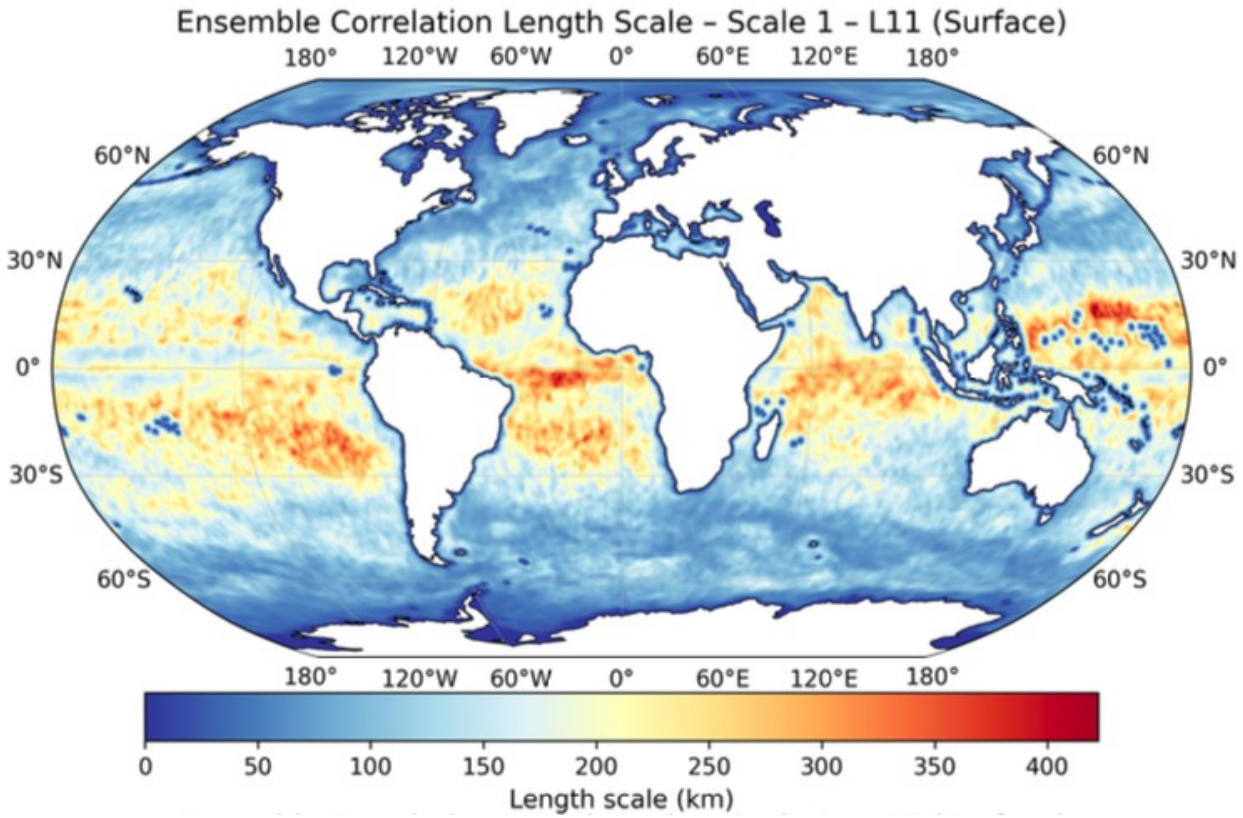
Small-scale term



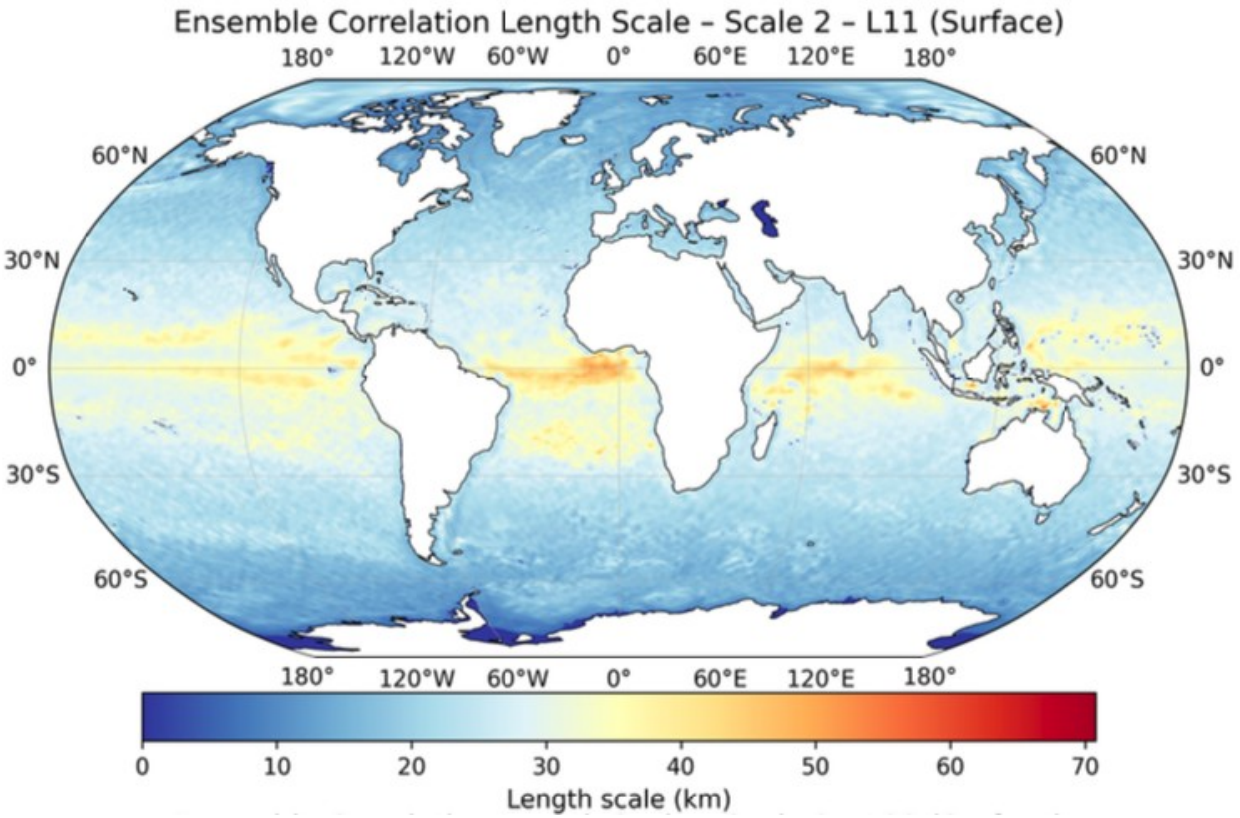
Scale-dependent climatological (DJF) zonal correlation length scales for temperature

Filtering length scale = β x local horizontal grid resolution, with β = 3

Large-scale term

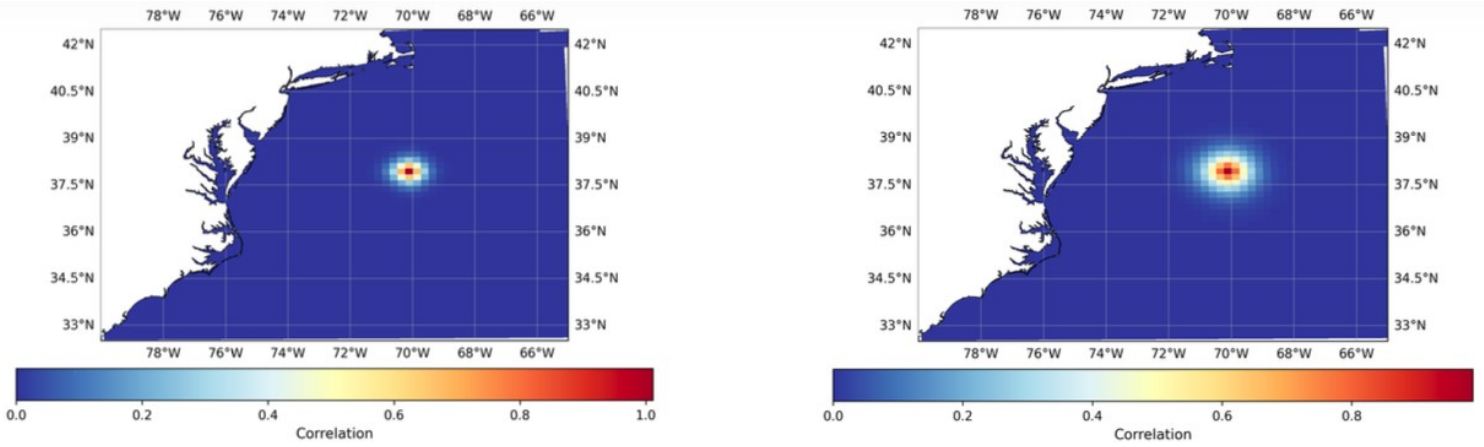


Small-scale term



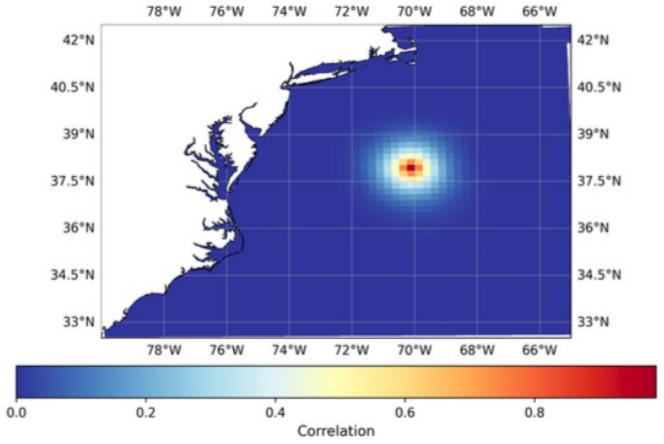
Temperature correlations

Filtering length scale = β x local horizontal grid resolution, for different values of β

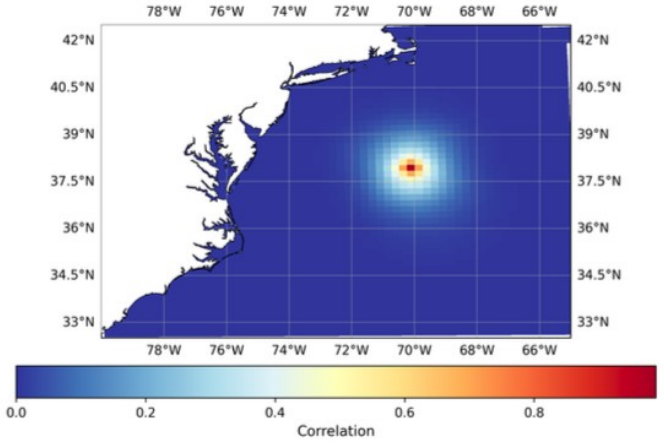


(a) Single-scale T–T correlation.

(b) Two-scale T–T correlation for $\beta_2 = 2$.

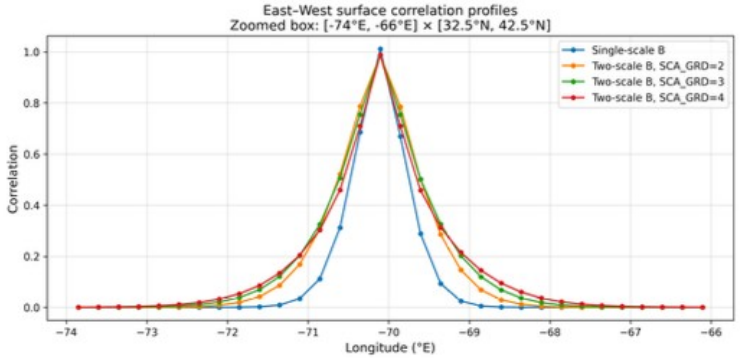


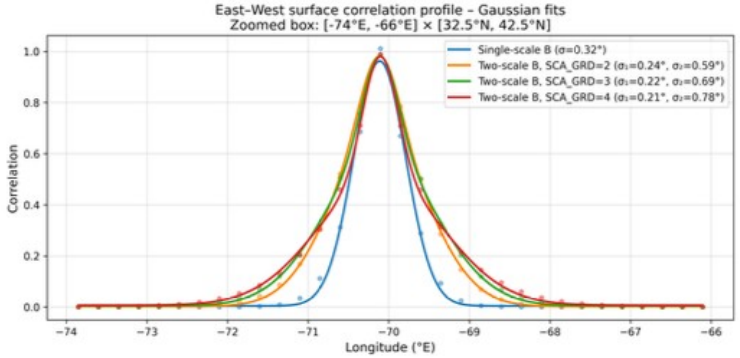
(c) Twp-scale T–T correlation for $\beta_2 = 3$.



(d) Two-scale T–T correlation for $\beta_2 = 4$.

East-west cross-section





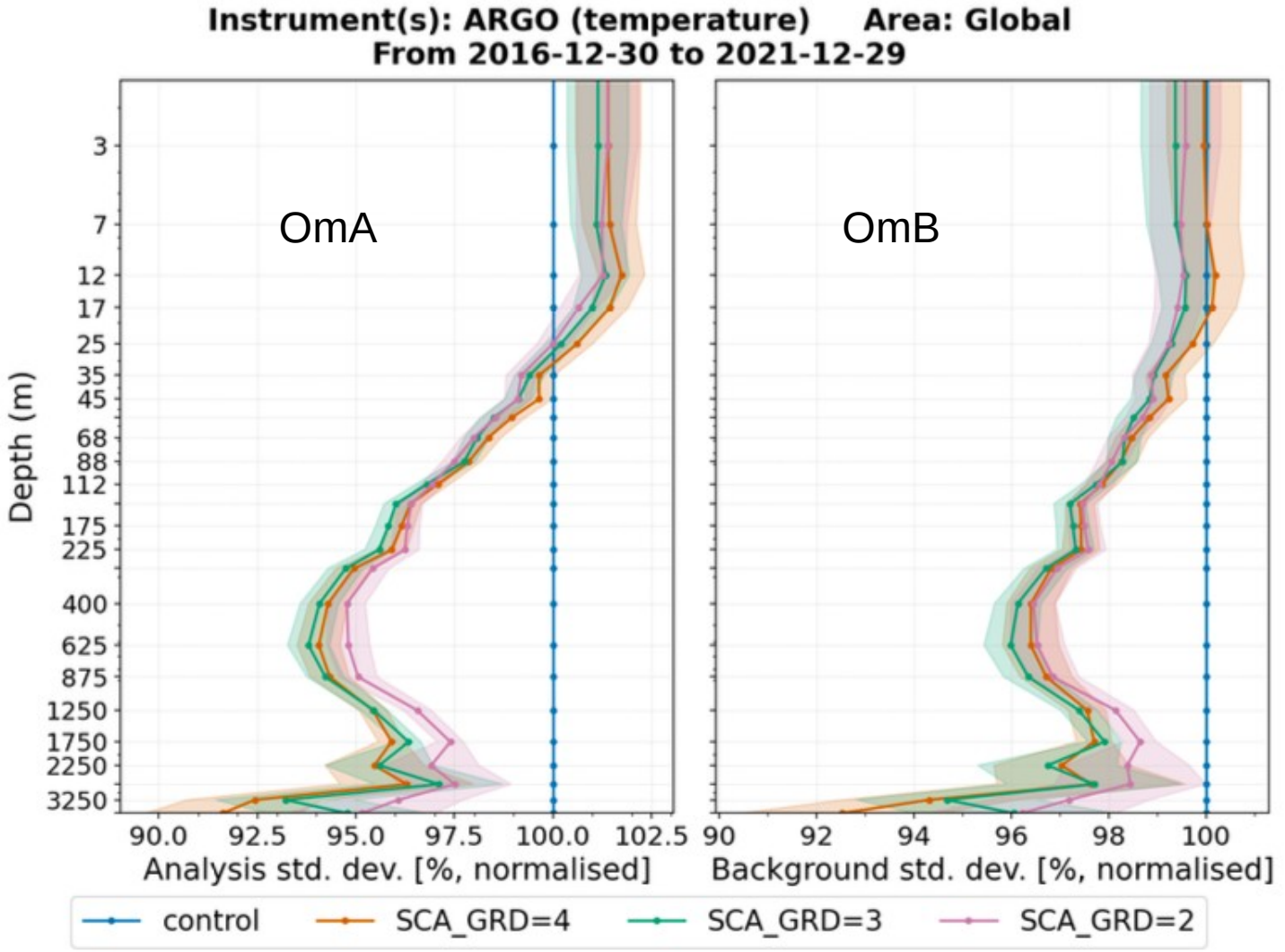
Gaussian fit

Globally averaged normalised std deviations of OmA and OmB for temperature

Experimental period: 2016-2021

Filtering length scale
= β x horizontal grid resolution

β	Experiment
0	control
2	SCA_GRD2
3	SCA_GRD3
4	SCA_GRD4

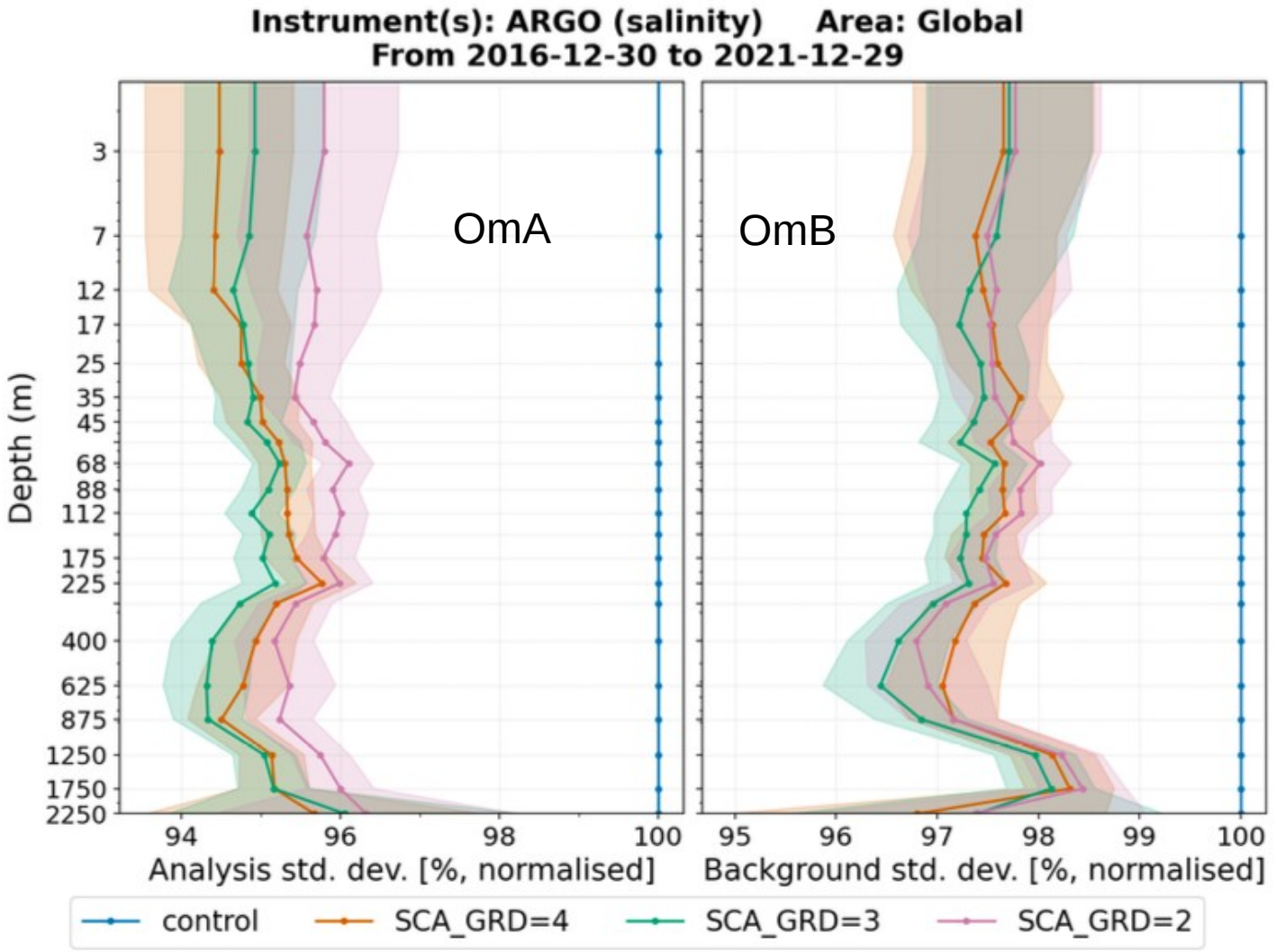


Globally averaged normalised std deviations of OmA and OmB for salinity

Experimental period: 2016-2021

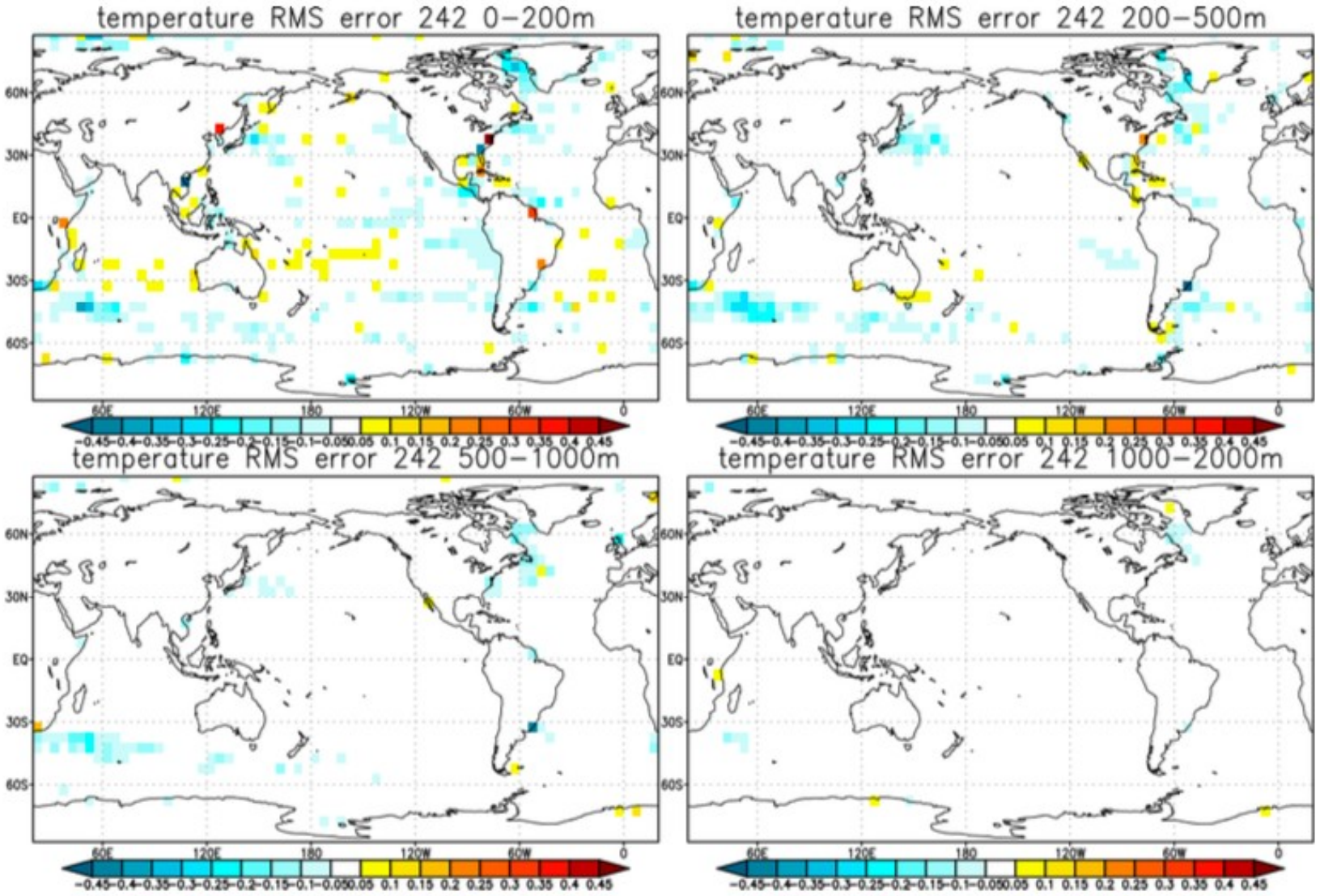
Filtering length scale
= β x horizontal grid resolution

β	Experiment
0	control
2	SCA_GRD2
3	SCA_GRD3
4	SCA_GRD4



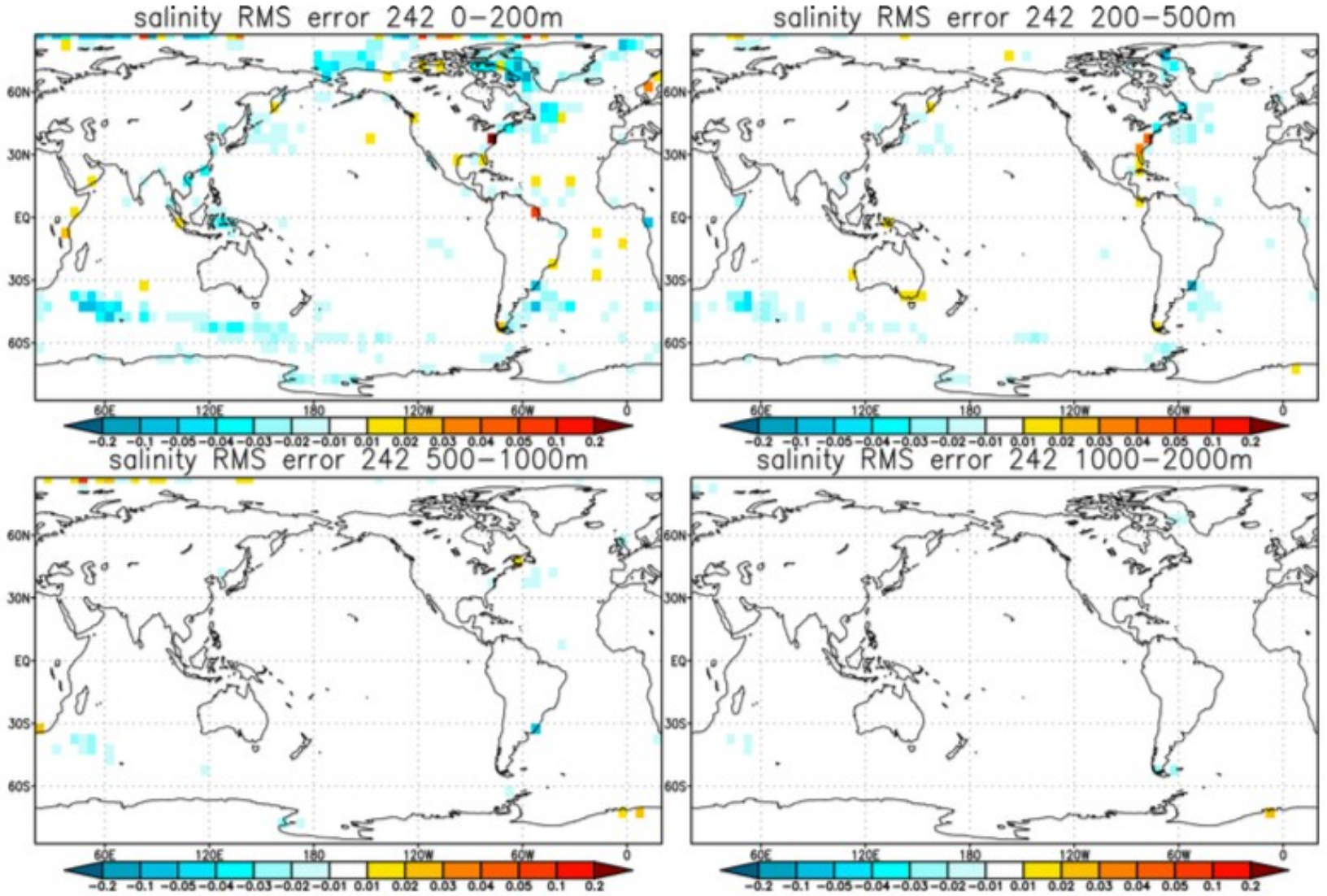
RMS error between SCA_GRD3 and control for temperature

Experimental period: 2016-2021



RMS error between SCA_GRD3 and control for salinity

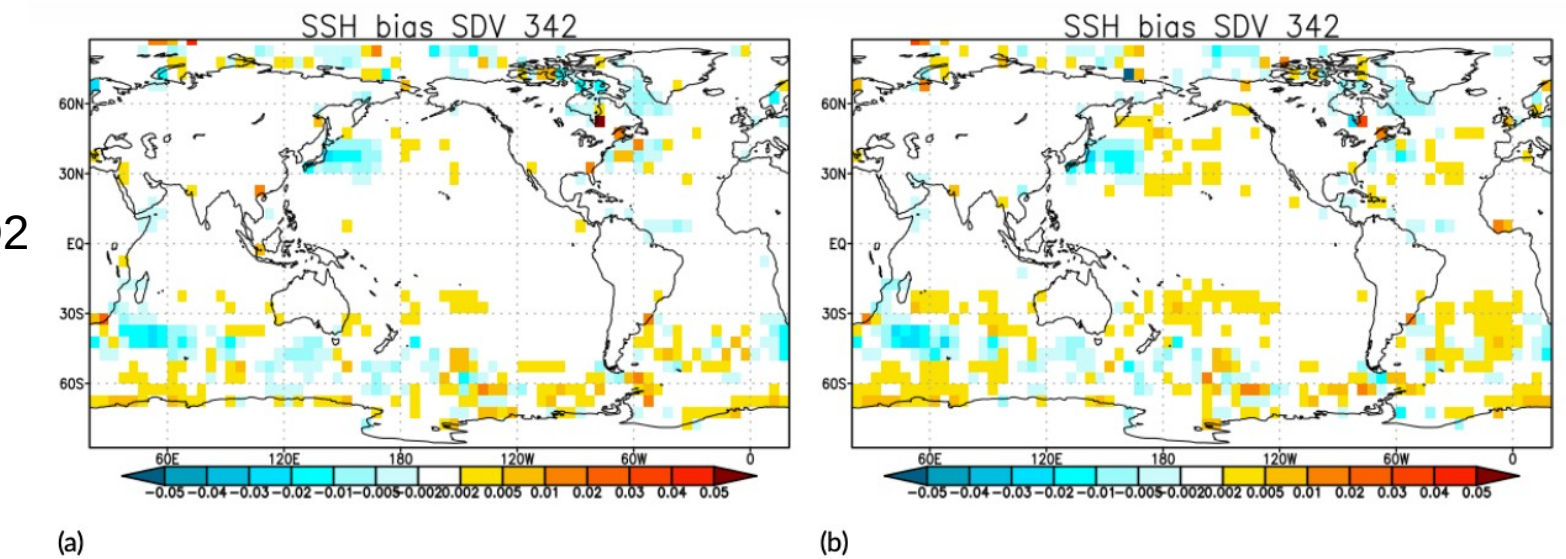
Experimental period: 2016-2021



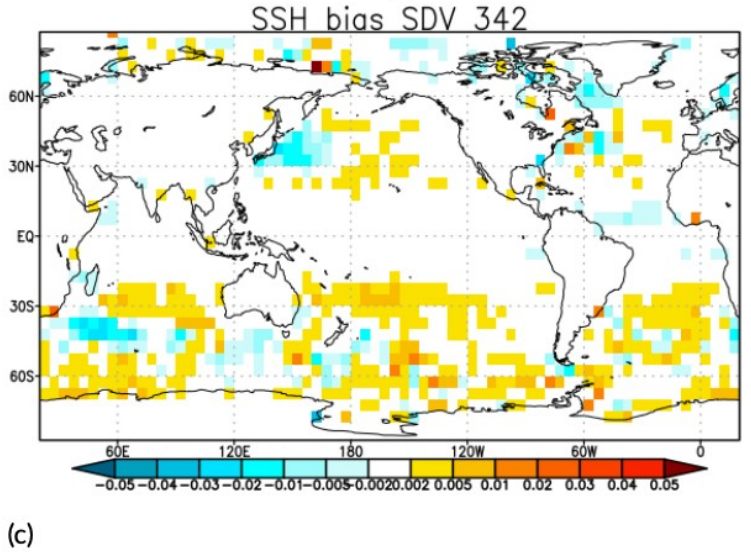
Std dev error between SCA_GRDx and control for SSH

Experimental period: 2016-2021

SCA_GRD2



SCA_GRD3



SCA_GRD4

(cf. results from Mignac et al. (2025))

Summary and outlook

- Multi-scale **B** results are positive for temperature and (especially) salinity; mixed for SSH.
- Better treatment of errors and biases in the MDT and SLA is required (bias correction; account for correlated errors – see O. Goux's presentation).
- More impact from multi-scale **B** expected with higher resolution configurations (1/12^{o} global; 1.5 km NWS regional).
- Improve computational efficiency using multi-resolution diffusion operator.
- Explore vertical as well as horizontal scale-separation.
- Generalise to flow-dependent, multi-scale **B** formulations (scale-dependent std deviations; anisotropic diffusion tensors).
- Hybridise with SDL.

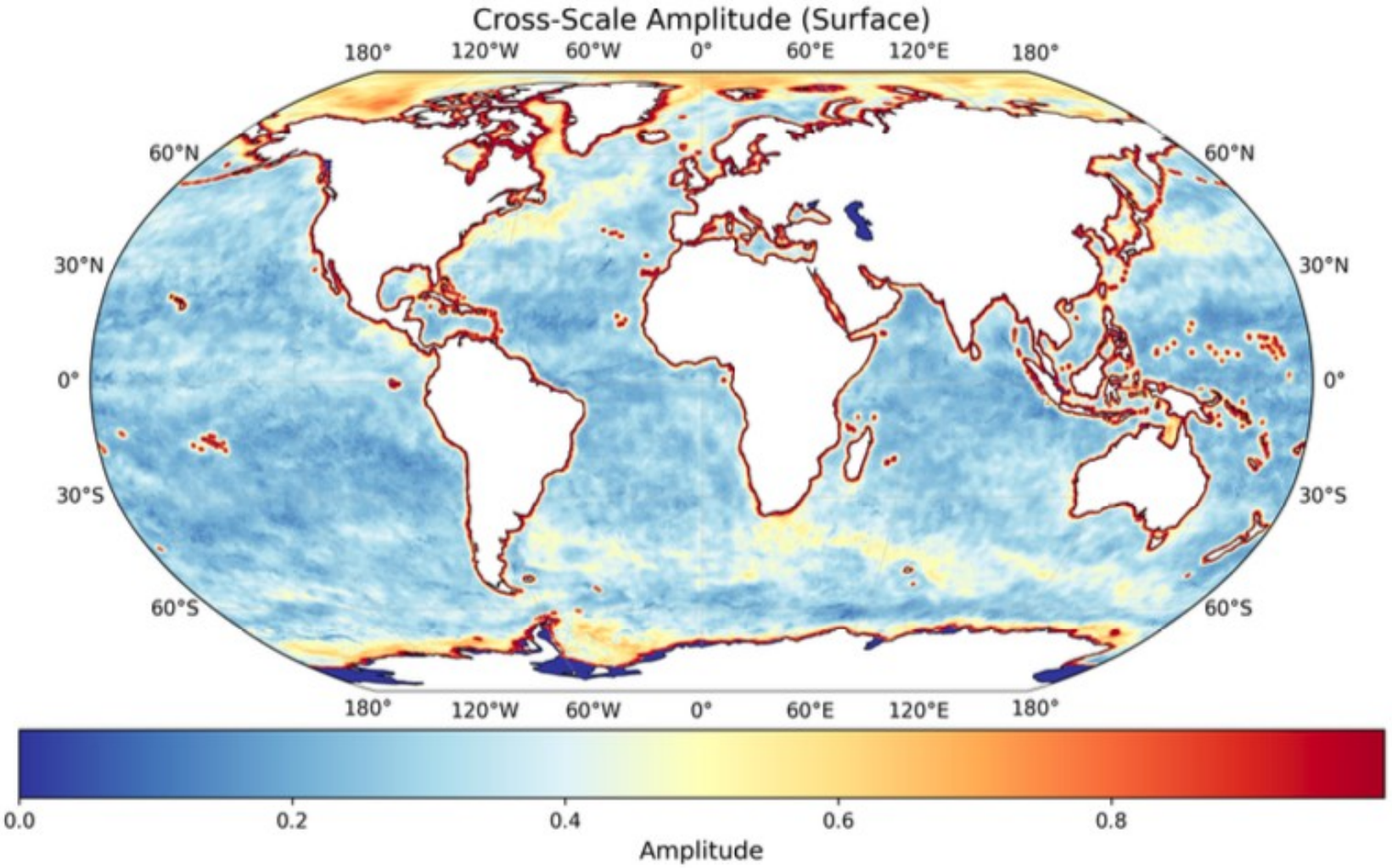
References

1. Buehner, M. and A. Shlyaeva, 2015: Scale-dependent background-error covariance localisation, *Tellus*, **67**, 28027.
2. Chrust, M., Weaver, A. T., Browne, P., Zuo, H. and M. A. Balmaseda, 2025: Impact of ensemble-based hybrid background-error covariances in ECMWF's next-generation ocean reanalysis system. *Quarterly Journal of the Royal Meteorological Society*, **151**, e4914.
3. Mignac, D. and co-authors, 2025: Improvements to the Met Office's global ocean-sea ice forecasting system including model and data assimilation changes. *Geoscientific Model Development*, **18**, 3405-3425.
4. Mirouze, I., Blockley, E. W., Lea, D. J., Martin, M. J. and M. J. Bell, 2016: A multiple length-scale correlation operator for ocean data assimilation. *Tellus A*, **68**, 29744.
5. Weaver, A. T. and M. Chrust, 2026: A scale-dependent background-error covariance model for ensemble-variational data assimilation, with application to ECMWF's ocean reanalysis system. Submitted to *Quarterly Journal of the Royal Meteorological Society.*
6. Weaver, A. T. and I. Mirouze, 2013: On the diffusion equation and its application to isotropic and anisotropic correlation modelling in variational assimilation. *Quarterly Journal of the Royal Meteorological Society*, **139**, 242-260.
7. Zuo, H. and co-authors, 2026: ORAS6: The new ECMWF ensemble ocean and sea-ice reanalysis-analysis system and impact on coupled forecasts. *In preparation*.

Extra slides

Amplitude of the cross-scale covariance term $\mathbf{C}_{ij}$, $i \neq j$

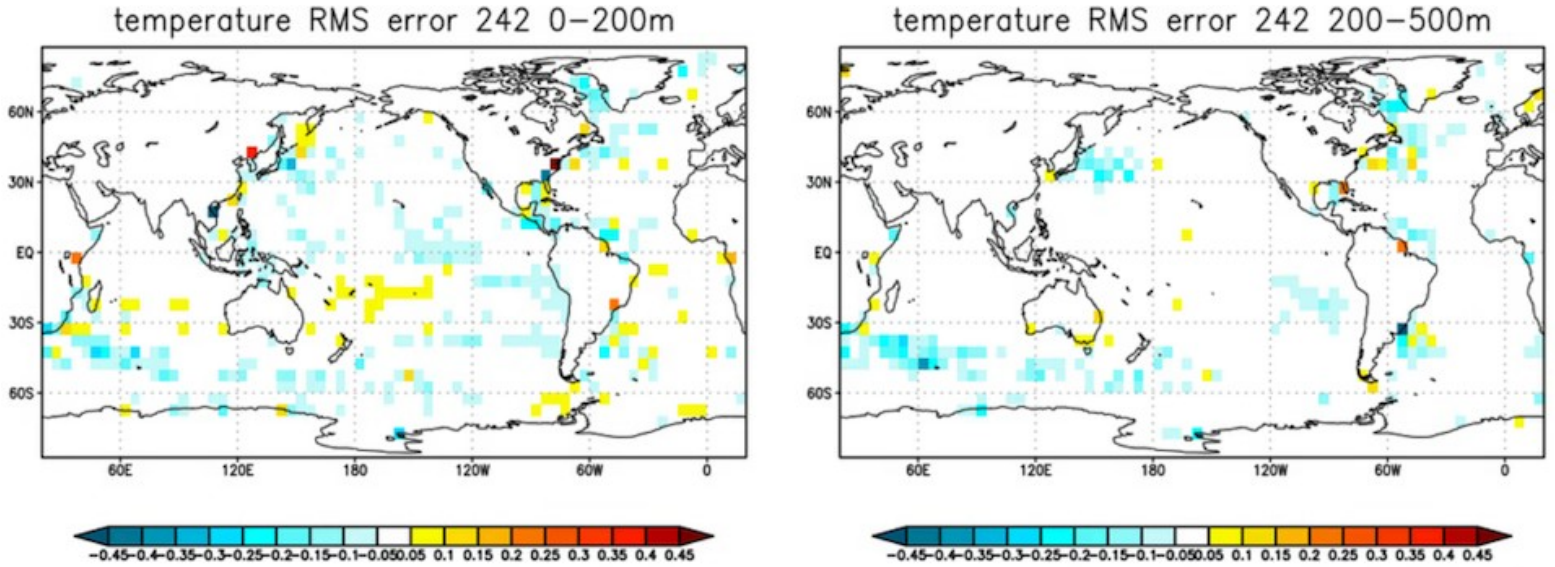
Required for estimating global normalisation factors



RMS error between SCA_GRD3 and control for temperature

Experimental period: 2016-2021

Climatological scale-dependent std deviations



Flow-dependent scale-dependent std deviations

