Hybrid covariance super-resolution data assimilation

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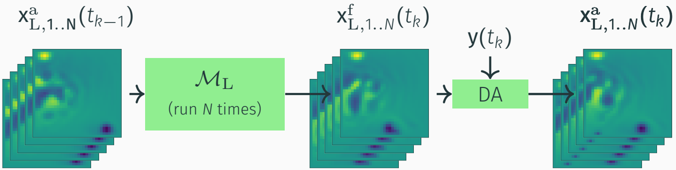
Overview

Objectives

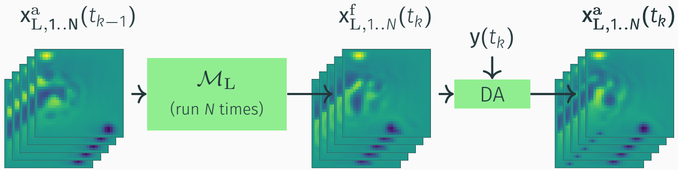
The objectives are twofold:

1. Emulating a High-Resolution (HR) EnKF while running the forecast step with a Low-Resolution (LR) model
   $\Rightarrow$ reduction of the computational cost of the EnKF
2. Taking advantage of HR observations and reducing LR model bias

EnKF - Low Resolution (EnKF-LR)



EnKF - Low Resolution (EnKF-LR)



	EnKF-LR		
Observation error	High✘		
High-resolution processes	Poorly resolved✘		
Computational cost	Low✔		
Ensemble size	Large✔		
Error to the true $\mathbf{P}^{\mathrm{f}}$	Large✘		

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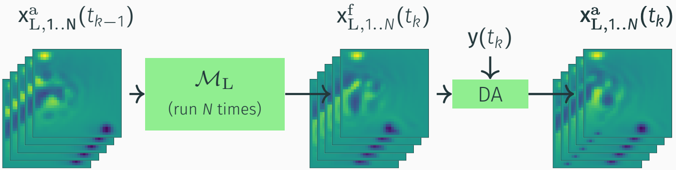
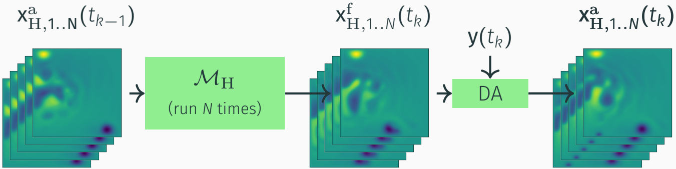


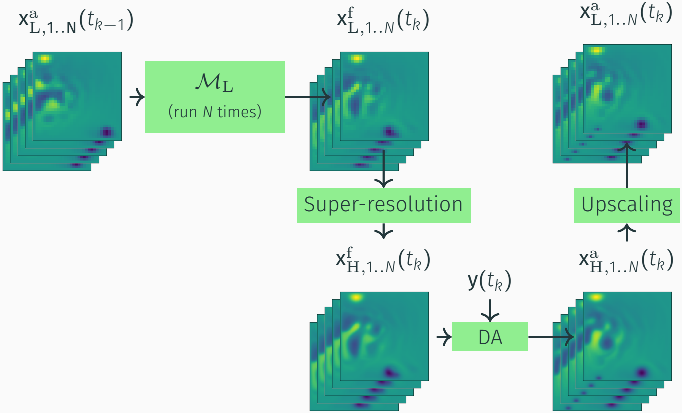
EnKF - High Resolution (EnKF-HR)



	EnKF-LR	EnKF-HR	
Observation error	High✔	Low✔	
High-resolution processes	Poorly resolved✔	Resolved✔	
Computational cost	Low✔	High, $\mathcal{O}(n^3)$✔	
Ensemble size	Large✔	Small✔	
Error to the true $\mathbf{P}^{\mathrm{f}}$	Large✔	Small✔	

Motivation and method

EnKF - Super-resolution data assimilation (SRDA)



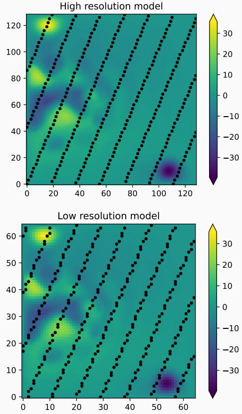
	EnKF-LR	EnKF-HR	SRDA
Observation error	High✔	Low✔	Low✔
High-resolution processes	Poorly resolved✔	Resolved✔	Emulated✔
Computational cost	Low✔	High, $\mathcal{O}(n^3)$✔	Low✔
Ensemble size	Large✔	Small✔	Large✔
Error to the true $\mathbf{P}^{\mathrm{f}}$	Large✔	Small✔	Medium✔

Model used

- Model used: Quasi-geostrophic model [Sakov and Oke, 2008]

Configuration	State size	Cost
HR	129×129	C
LR	65×65	$C/8$

- Observations:
  - True value perturbed by a Gaussian noise of standard deviation 2
  - Available every $\Delta t = 12$
  - Located along simulated satellite tracks (black dots on the figures)
  - Note the representativeness errors.



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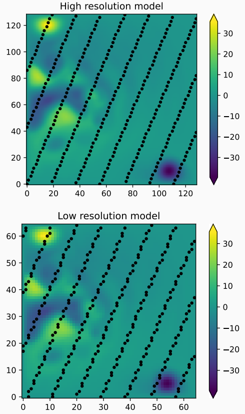


Super-resolution: downscaling operator

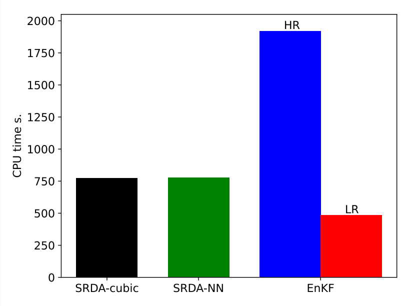
▶ A simple cubic spline interpolation (cubic)

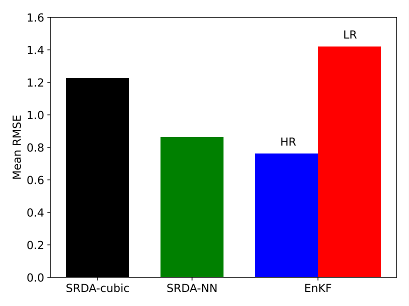
▶ A neural network (NN) $\Rightarrow$ corrects the LR model error

Overview

DA experiments at constant computational cost of assimilation

- Synthetic experiments with 500 assimilation cycles and 25 members





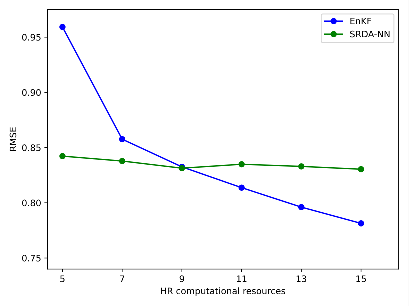
- SRDA-NN provides a good compromise between RMSE and computational cost.

DA experiments at a constant computational cost

Trade-off: 1 HR member $\approx$ 8 LR members (integration time)

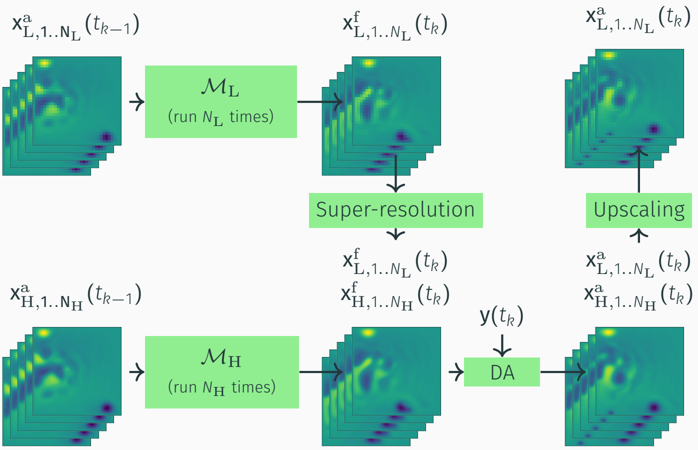
Design of the experiments:

- Twin experiments with 500 assimilation cycles;
- Compared performance at equivalent computational cost of $\approx 5, 7, \ldots, 15$ HR members $\rightarrow$ with $40, 48, \ldots, 120$ LR members for the SRDA
- Localization and inflation tuned to optimal performance



- The SRDA improves the mean RMSE for limited computational resources
- For larger resources, the error from the emulator is the bottleneck
- Can we take the best of both worlds?

Overview



	EnKF-LR	EnKF-HR	SRDA	Hybrid SRDA
Observation error	High✔	Low✔	Low✔	Low✔
HR processes	Poorly resolved✔	Resolved✔	Emulated✔	Emulated (LR)✔/ resolved (HR)✔
Computational cost	Low✔	High, $\mathcal{O}\left(n^3\right)$✔	Low✔	Customizable(✔–✔)
Ensemble size	Big✔	Small✔	Big✔	Big✔
Error to the true P^f	Large✔	Small✔	Medium✔	Customizable(✔–✔)

Hybrid covariance Super-Resolution Data Assimilation

The covariance matrix $\mathsf{P}^{\mathrm{f}}_{\mathrm{h}}$ in the Hybrid SRDA is a linear combination of:

- $\mathsf{P}^{\mathrm{f}}_{\mathrm{HR}}$ computed from the HR ensemble;
- $\mathsf{P}^{\mathrm{f}}_{\mathrm{LR}}$ computed from the LR ensemble **downscaled** to the HR grid:

$$\mathsf{P}^{\mathrm{f}}_{\mathrm{h}} = (1-\alpha)\mathsf{P}^{\mathrm{f}}_{\mathrm{HR}} + \alpha \mathsf{P}^{\mathrm{f}}_{\mathrm{LR}}, \qquad 0 \leq \alpha \leq 1. \tag{1}$$

- $\alpha = 0$ full HR case $\rightarrow$ EnKF-HR
- $\alpha = 1$ full LR case $\rightarrow$ EnKF-LR
- downscaling method "cubic spline interpolation" – Hybrid SRDA cubic
  $\Rightarrow$ Mixed-resolution ensemble data assimilation [Rainwater and Hunt, 2013].
- Results computed over the HR ensemble unless otherwise stated.
- Optimal α: match between the spread and the RMSE of the HR ensemble [Anderson and Anderson, 1999].

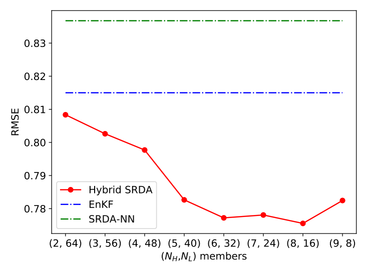
Fixed integration cost: trade-off HR/LR ensemble sizes experiments

Ensemble integration strongly dominates the cost of a data assimilation system

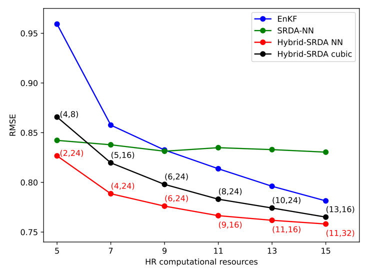
Trade off: 1 HR member $\approx$ 8 LR members (integration time)

Design of the experiments: Twin experiments (500 assim. cycles) with parameters and hybrid coefficients optimally tuned

- **Figure (a):**
  - **EnKF-HR** with $N_{\mathrm{H}} = 10$ members, **SRDA-NN** with $N_{\mathrm{L}} = 80$ members
  - At equivalent computational cost, **Hybrid SRDA-NN** with $(N_{\mathrm{H}}, N_{\mathrm{L}}) = (2, 64), (3, 56), \ldots, (9, 8)$
- **Figure (b):**
  - We repeat the comparison for HR resources ranging from 5 to 15 members



(a)



(b)

Overview

Conclusions/perspectives

Main result

▶ The **Hybrid SRDA** outperforms the SRDA, the EnKF and the Mixed-resolution ensemble data assimilation at an equivalent computational cost.

▶ The method is highly customizable and makes optimal use of the computational resources available at hands.

▶ The effort to implement (hybrid)-SRDA in an existing DA system is minimal

- No need to modify existing models
- Minor modifications to the DA code in case of Hybrid SRDA

Conclusions/perspectives

Main result

- The **Hybrid SRDA** outperforms the SRDA, the EnKF and the Mixed-resolution ensemble data assimilation at an equivalent computational cost.
- The method is highly customizable and makes optimal use of the computational resources available at hands.
- The effort to implement (hybrid)-SRDA in an existing DA system is minimal
  - No need to modify existing models
  - Minor modifications to the DA code in case of Hybrid SRDA

Perspectives

- Hybrid SRDA paper in preparation
- Application of SRDA in TOPAZ system $\longrightarrow$ post-doc Nansen Center supervised by Julien Brajard & Laurent Bertino
- Application to the Norwegian Climate Prediction Model (NorCPM) within the project EU-Impetus4Change
- Do not throw away your old coarse-resolution models!

SRDA (only!) paper available on *Ocean Dynamics*!

https://link.springer.com/article/10.1007/s10236-022-01523-x

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Tellus, Series A: Dynamic Meteorology and Oceanography, 60 A(2):361–371.

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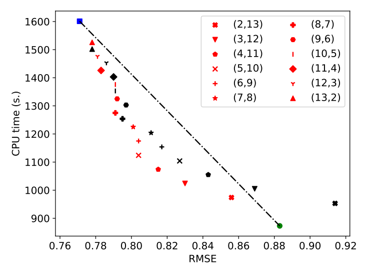
Fixed assimilation cost: fixed ensemble size & computational efficiency

- Twin experiments with 500 assimilation cycles
- Experiments at fixed ensemble size: (N_H, N_L) such that $N_H + N_L = 15$
- Optimal inflation and localization



Hybrid SRDA-cubic

EnKF

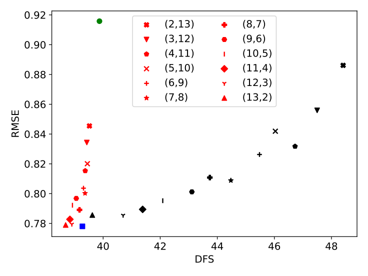
Hybrid SRDA-NN

SRDA-NN

- The method is cost effective if it is under the black dashed line;

Performance in terms of degrees of freedom of the signal

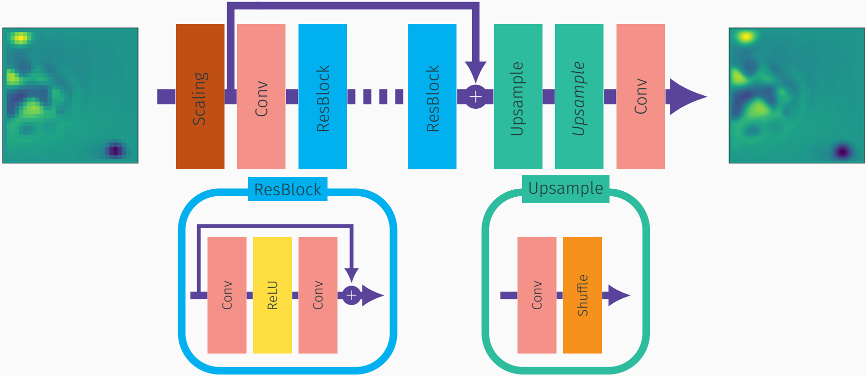
▶ **We seek a system that achieves the lowest error, doing minimal corrections.**
▶ DFS = Tr (**KH**) ⇒ quantify the number of degrees of freedom reduced from the ensemble. [Cardinali *et al.*, 2004].
▶ Example of DFS with the different method at equivalent 15 HR members



Hybrid SRDA-cubic
EnKF
Hybrid SRDA-NN
SRDA-NN

▶ The Hybrid SRDA-NN yields lower DFS (assimilation updates) than the Hybrid SRDA-cubic.

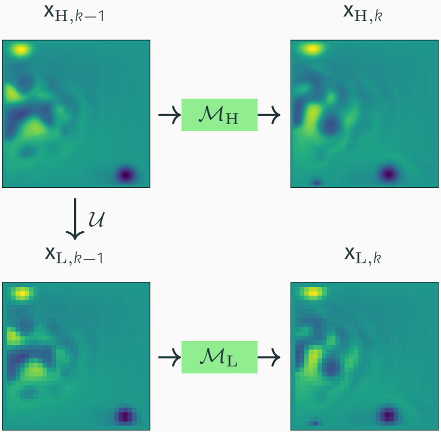
Setup of the neural network



Architecture of the enhanced deep super-resolution network (EDSR) [Lim et al., 2017]

Training set for the neural network

- Run one simulation of the HR model.
- Assemble matching pairs of (U)LR and HR states: $(\mathsf{x}_{\mathrm{L},k}, \mathsf{x}_{\mathrm{H},k})$



$\mathcal{U}$: Upscaling (subsampling operator)

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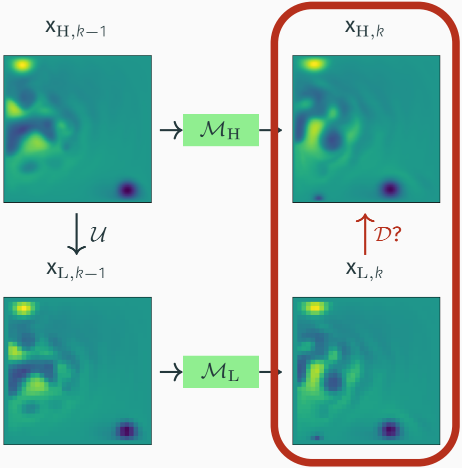


$\mathcal{U}$: Upscaling (subsampling operator)
$\mathcal{D}$: Downscaling (Neural network)

- Number of pairs: 10,000
- 8000 for training / 2000 for validation
- Architecture of the enhanced deep super-resolution network (EDSR) [Lim et al., 2017]
- Training: minimization of the mean absolute error

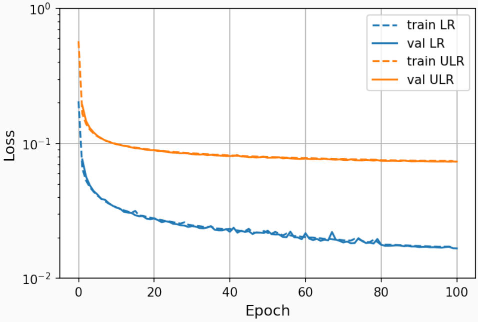
Training of the neural network

Minimize the mean absolute error (MAE):

$$L(\mathbf{w}) = \sum_{k=1}^{K} \sum_{i=1}^{S} \left| \mathcal{D}(\mathbf{x}_{\mathrm{L},k})_i - x_{\mathrm{H},k,i} \right| ,$$

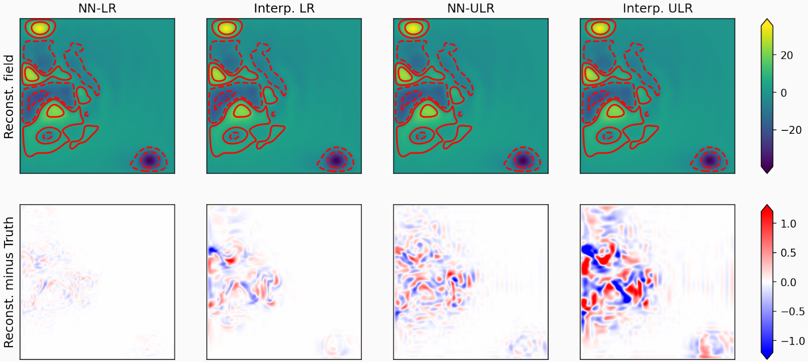
i:	the pixel index
S:	size of the state (129×129)
K:	size of the training set (K=8000)
$\mathbf{w}$:	weights of the neural network ($\sim 20,000$)

Training curve



Downscaling performance

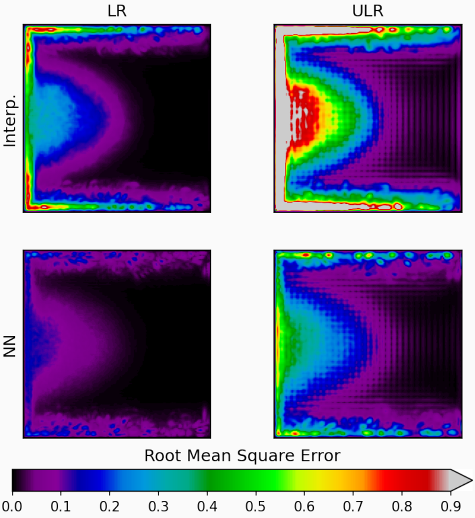
- Illustration with one typical sample



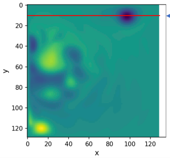
red lines: Contour of the true HR state

Downscaling performance (2)

- Score on the validation dataset

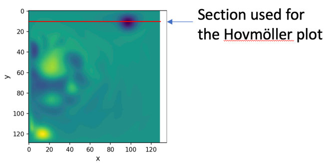


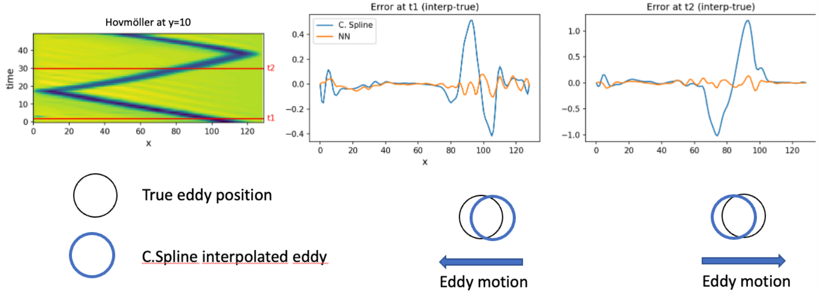
Model error correction



Section used for the Hovmöller plot

Model error correction



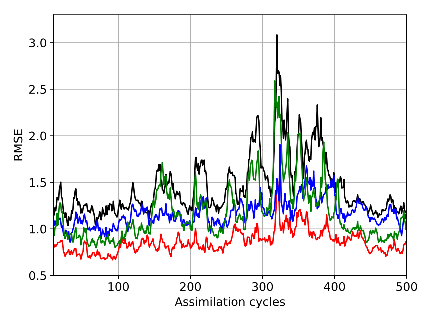


- Eddy propagation slower in the LR model
- The NN is smart enough to learn that

Reformulating the SRDA as a LR scheme

- We can reformulate the SRDA into LR EnKF equations so that we can separate the contributions from:

  1. the model error correction;
  2. the super-resolution observation operator (representativeness).



- Model error correction improves performance during challenging events
- Super-resolution obs. operator reduces error over the whole period

EnKF-LR
SRDA only with model error correction
Complete SRDA-NN
SRDA only with the super-res. observation operator