Improving vortex position accuracy with a new multiscale alignment ensemble filter

Yue (Michael) Ying
NERSC

Jeffrey Anderson (*NCAR*), Laurent Bertino (*NERSC*)

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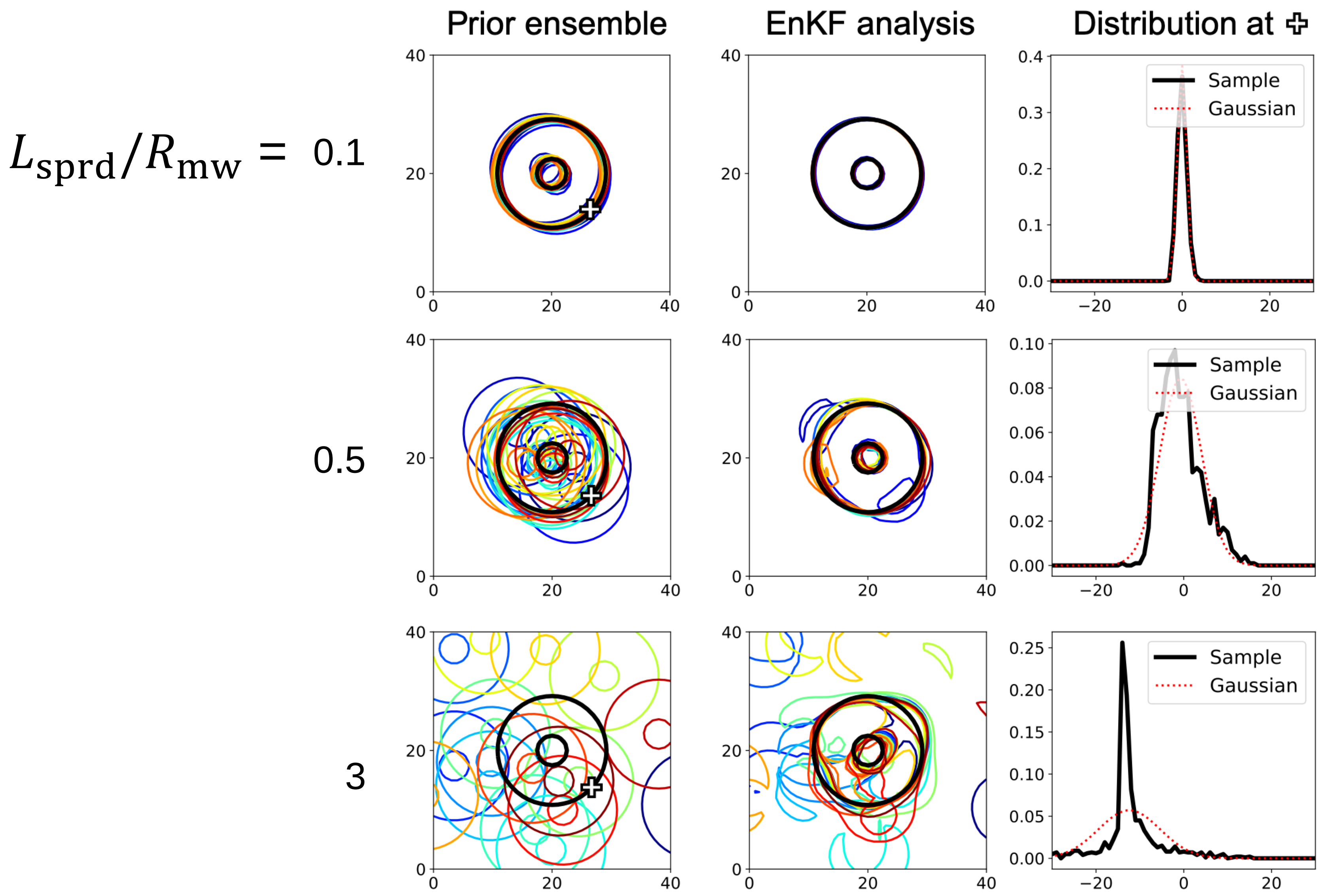
Nonlinearity due to vortex position errors



Contours of constant wind speed from vortices:
black: truth
colors: ensemble members

As position error L_{sprd} increases,

- error distribution becomes more non-Gaussian,
- EnKF analysis becomes more suboptimal

Data assimilation with position uncertainties

Error model: $\mathbf{X}^b = \mathbf{X}^* + \boldsymbol{\varepsilon}^d + \boldsymbol{\varepsilon}^r$

$\boldsymbol{\varepsilon}^d = \mathbf{X}^b - \mathbf{X}^b(\mathbf{q})$ displacement error

$\boldsymbol{\varepsilon}^r = \mathbf{X}^b(\mathbf{q}) - \mathbf{X}^* \sim \mathcal{N}[0, \mathbf{B}(\mathbf{q})]$

residual (amplitude) error

Bayesian formulation on posterior error distribution:

$$p(\mathbf{X}, \mathbf{q} | \mathbf{Y}) \propto p(\mathbf{Y} | \mathbf{X}, \mathbf{q})\, p(\mathbf{X} | \mathbf{q})\, p(\mathbf{q})$$

Cost function:

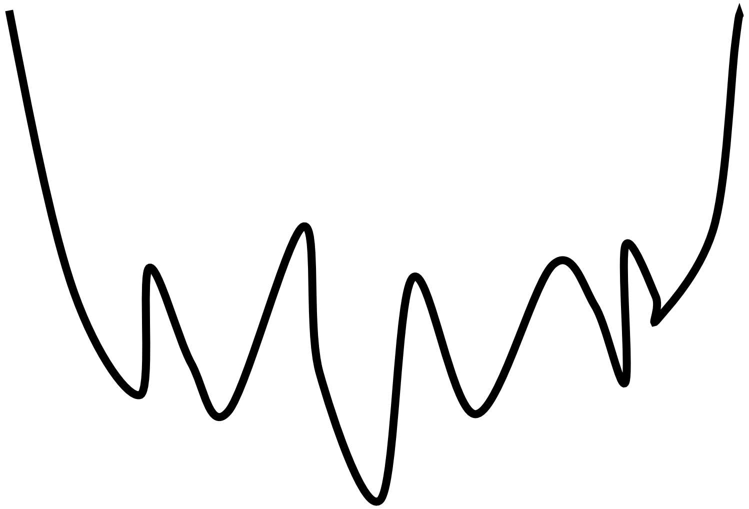
$$J(\mathbf{X}, \mathbf{q}) = \frac{1}{2} \|\mathbf{Y} - H[\mathbf{X}(\mathbf{q})]\|^2_{\mathbf{R}} + \frac{1}{2} \|\mathbf{X}(\mathbf{q}) - \mathbf{X}^b(\mathbf{q})\|^2_{\mathbf{B}(\mathbf{q})} + \frac{1}{2} \ln(|\mathbf{B}(\mathbf{q})|) + L(\mathbf{q})$$

Two-step solver: 1. derive displacement (update $\mathbf{q}$), 2. EnKF update (update $\mathbf{X}$)

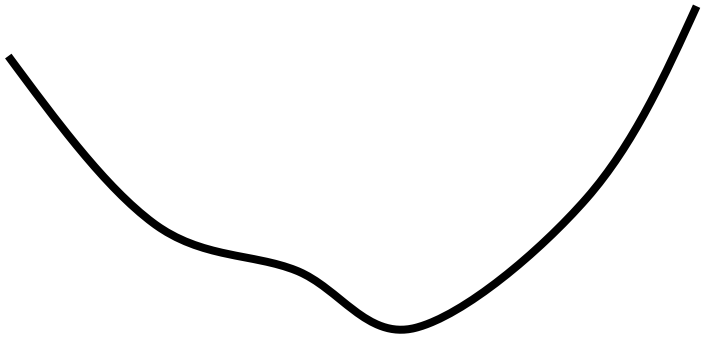
(Ravela et al. 2007, Nehrkorn et al. 2015)

Data assimilation with position uncertainties

Topography of $J(\mathbf{x},\mathbf{q})$:
Nonlinearity causes a lot of local minima and difficulty in reaching the global minimum through iterative solver



global minimum

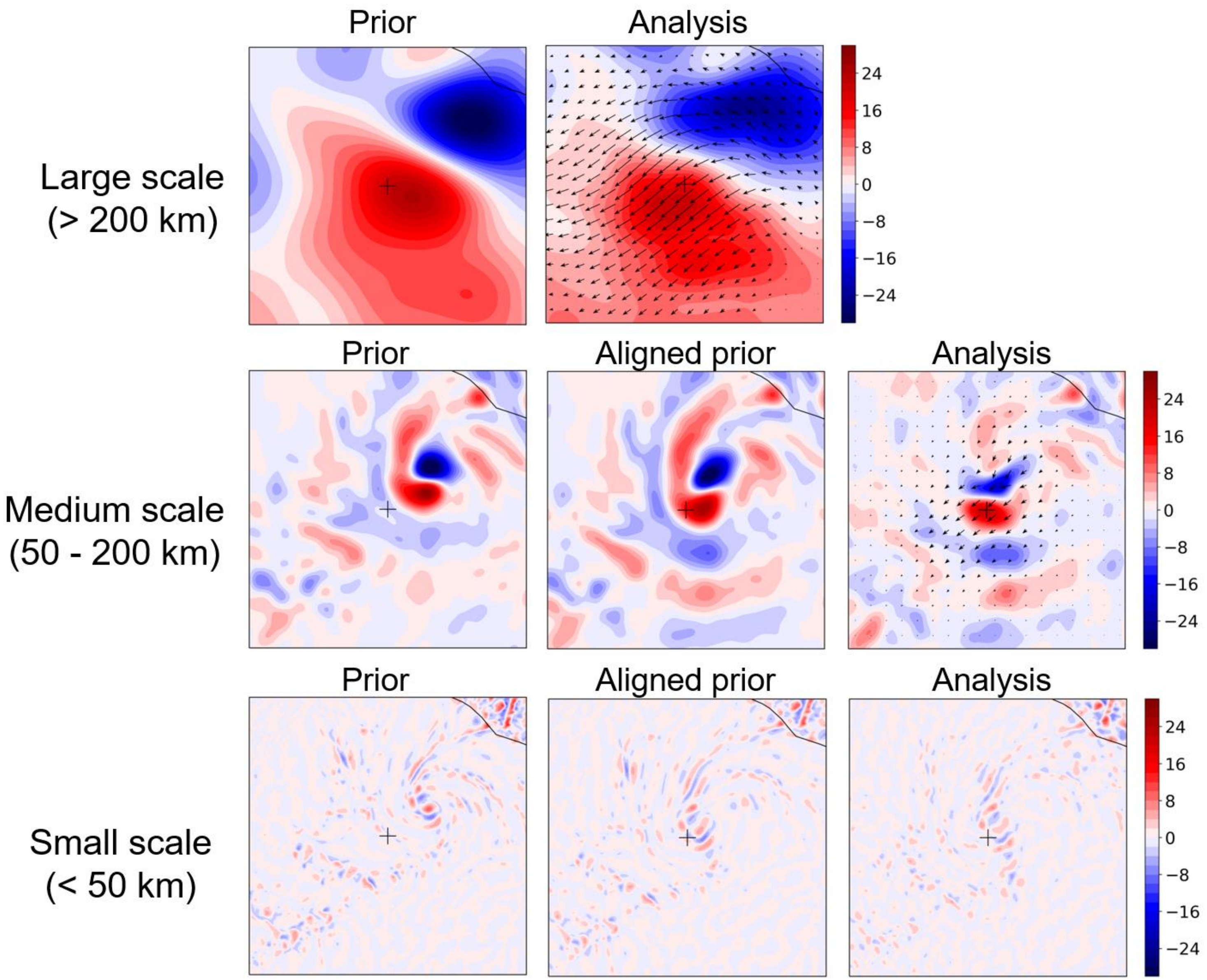


the same cost function but for lower resolution (larger scales)

Idea:

Use iterations over scale components (SCs) (outer loops in 4DVar)
The large-scale iteration skips local minima and save a lot of iterations in high-res space
Similar “multiscale idea” used in image processing (optical flow)

The multiscale alignment ensemble filtering idea



Example: Hurricane Patricia (2015)

blue/red shadings: *u*-wind for the *Ns* = 3 scale components.

vectors: the displacement vectors computed from the analysis increments

Iterate over scale components:

1. EnKF assimilate observations,
2. Find displacements (optical flows), which are applied to the smaller scales to align (precondition) the prior,
3. go to next scale …

The MSA EnKF algorithm

for s in $1, \ldots, N_s$ **do**

    $\mathbf{x}^b_{n,s} = \mathbf{F}_s \mathbf{x}_n$

    $\mathbf{y}^b_n = h(\mathbf{x}_n)$

    if decompose_obs **then**

        $\mathbf{y}^o_s = \mathbf{F}^o_s \mathbf{y}^o$

        $\mathbf{y}^b_{n,s} = \mathbf{F}^o_s h(\mathbf{x}_n)$

        $\mathbf{x}^a_{n,s} = \mathbf{x}^b_{n,s} + \mathbf{L}_s \circ \frac{\mathrm{cov}(\mathbf{x}^b_s, \mathbf{y}^b_s)}{\mathrm{cov}(\mathbf{y}^b_s, \mathbf{y}^b_s) + \sigma^2_{o,s}\mathbf{I}} \left(\mathbf{y}^o_s - \mathbf{y}^b_{n,s}\right)$

    else

        $\mathbf{x}^a_{n,s} = \mathbf{x}^b_{n,s} + \mathbf{L}_s \circ \frac{\mathrm{cov}(\mathbf{x}^b_s, \mathbf{y}^b)}{\mathrm{cov}(\mathbf{y}^b, \mathbf{y}^b) + \sigma^2_{o}\mathbf{I}} \left(\mathbf{y}^o - \mathbf{y}^b_{n}\right)$

    end if

    if $s < N_s$ **then**

        $\mathbf{q}_{n,s} = \underset{\mathbf{q}}{\mathrm{argmin}} \left\| \mathbf{x}^b_{n,s}(\mathbf{q}) - \mathbf{x}^a_{n,s} \right\|^2 + w \left\| \nabla \mathbf{q} \right\|^2$

        $\mathbf{x}_n \leftarrow \mathbf{x}_n(\mathbf{q}_{n,s}) + \mathbf{x}^a_{n,s} - \mathbf{x}^b_{n,s}(\mathbf{q}_{n,s})$

    else

        $\mathbf{x}_n \leftarrow \mathbf{x}_n + \mathbf{x}^a_{n,s} - \mathbf{x}^b_{n,s}$

    end if

end for

MSA (Ying 2019)

MSA-O (decompose_obs option added)

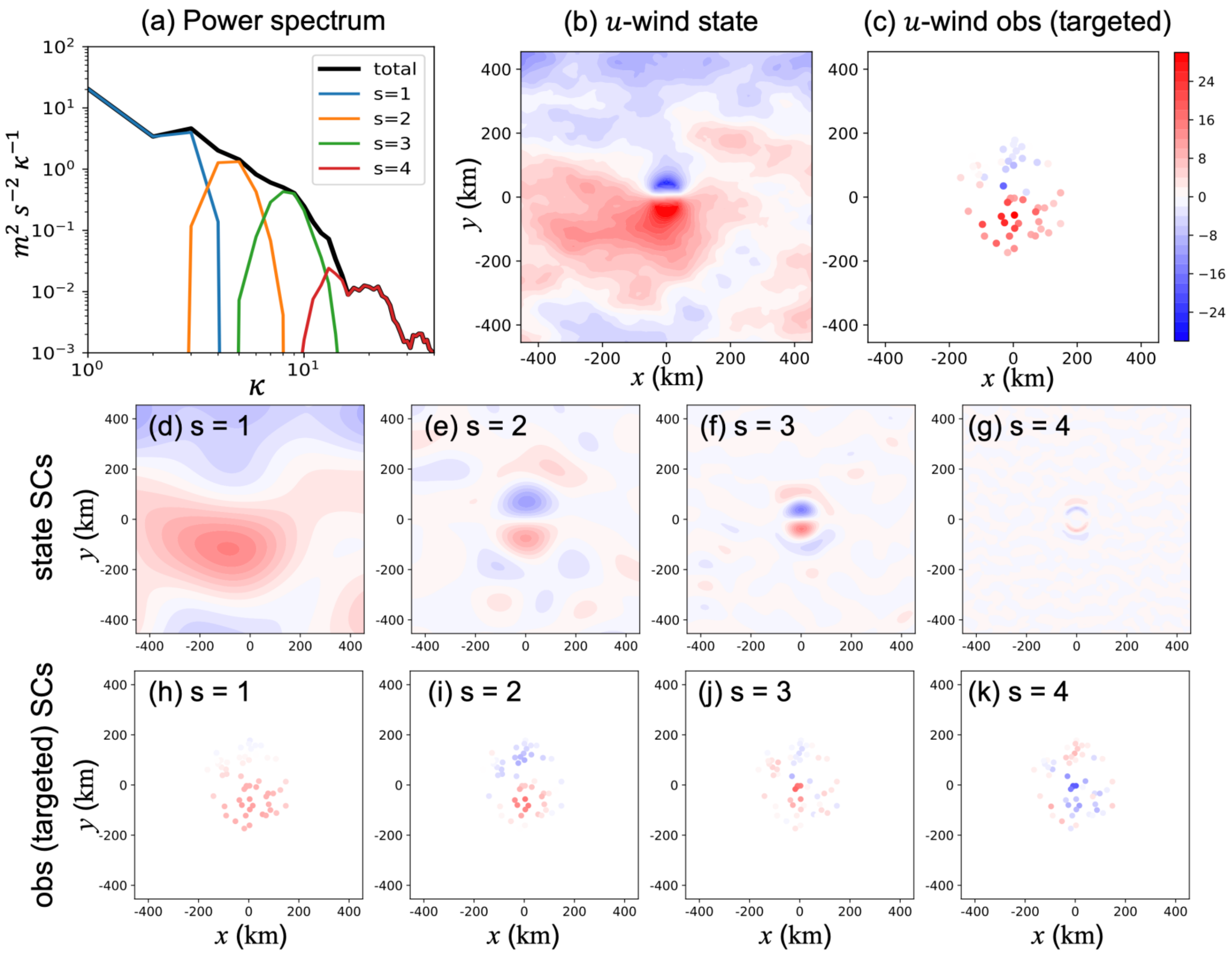
Filter update step

Alignment step

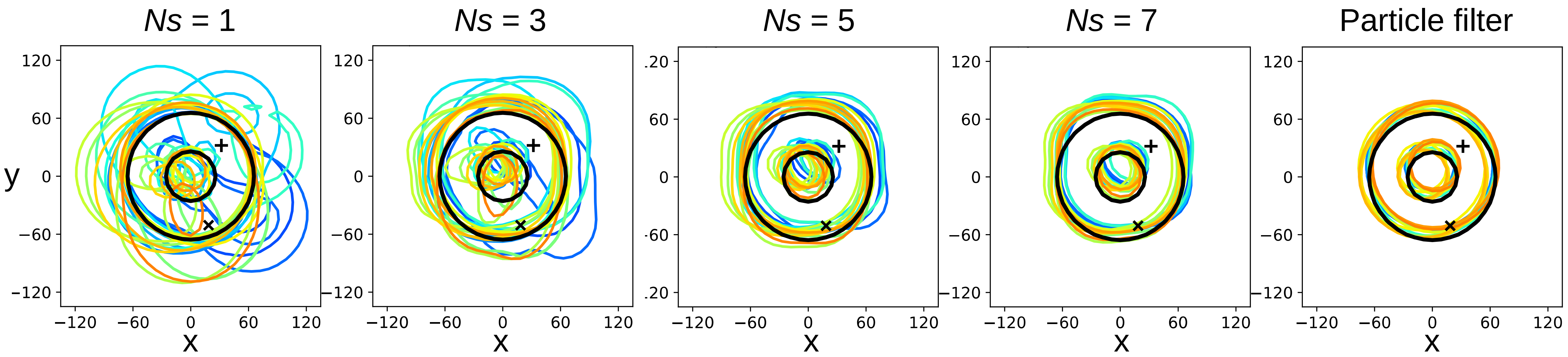
$n = 1, \ldots, N$ indexes ensemble members

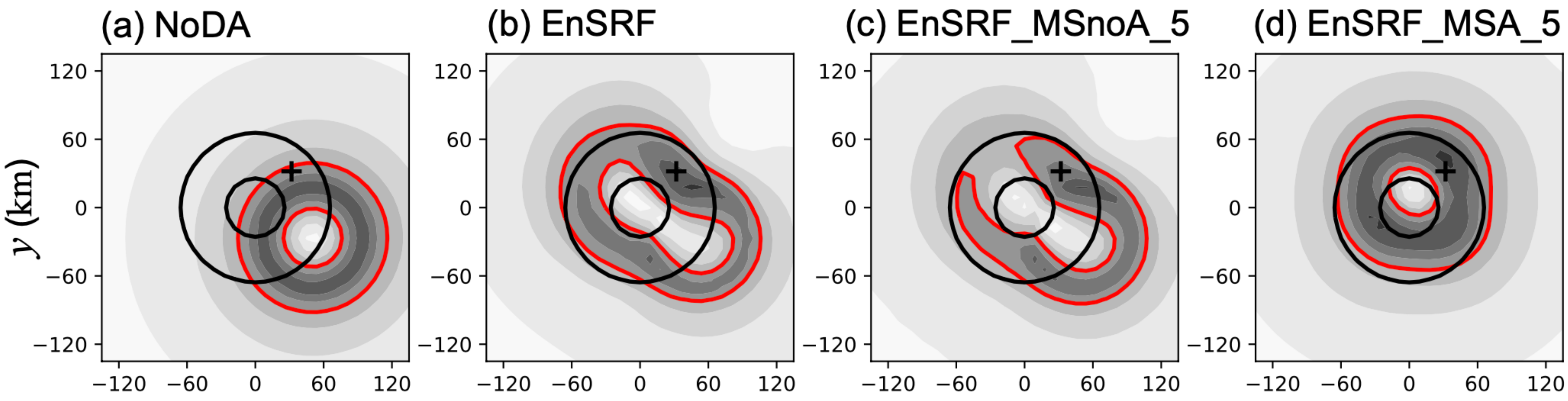
$s = 1, \ldots, Ns$ indexes scale components (SC)

Test case: 2D vortex embedded in background flow

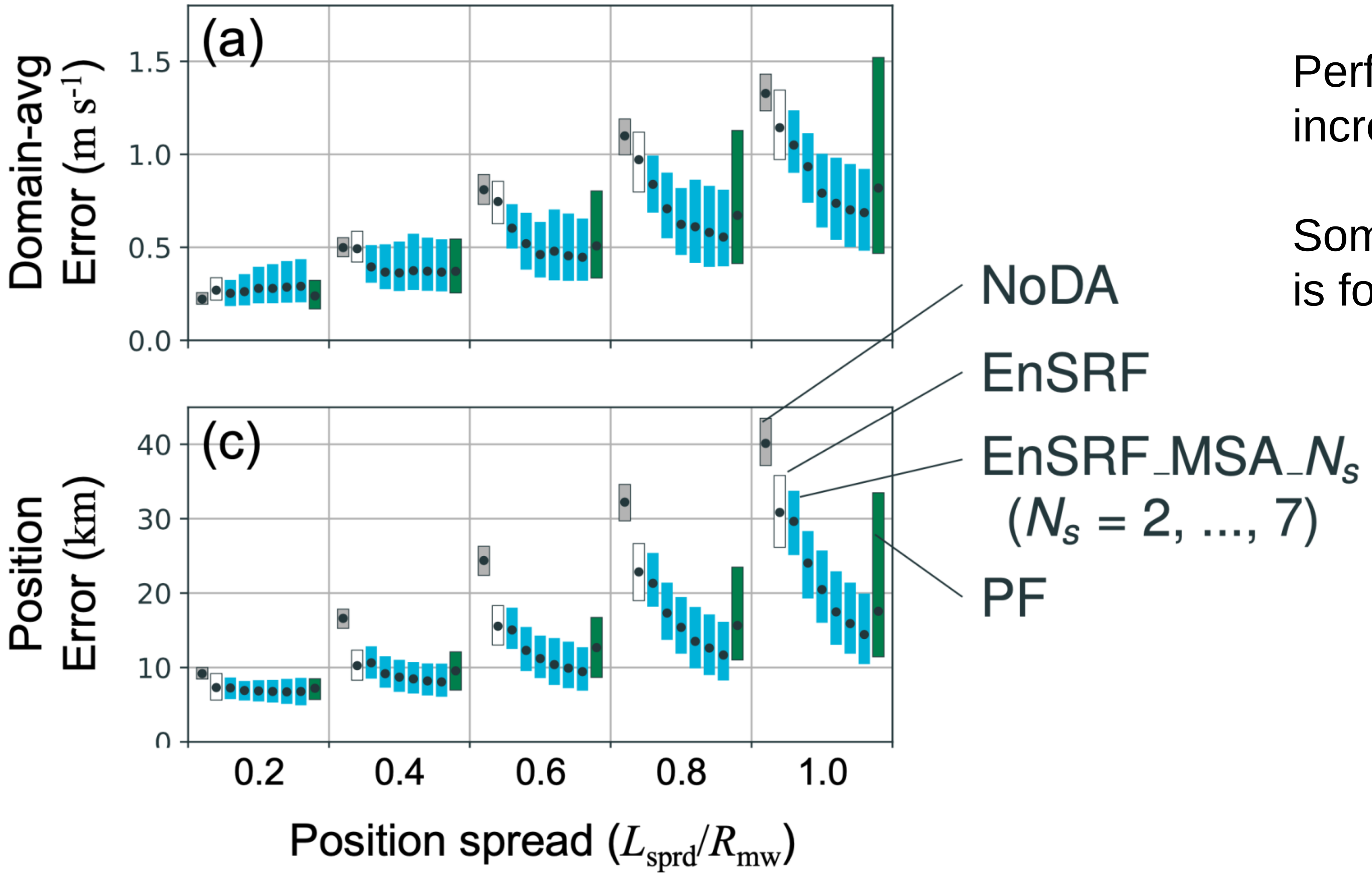


Asmptotic behavior as *Ns* increases



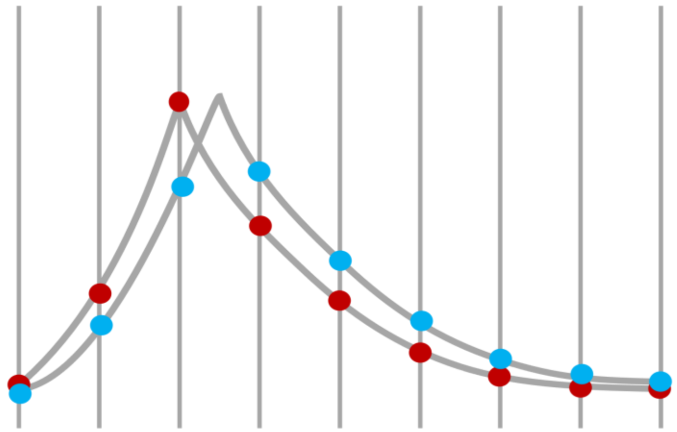


Asmptotic behavior as *Ns* increases



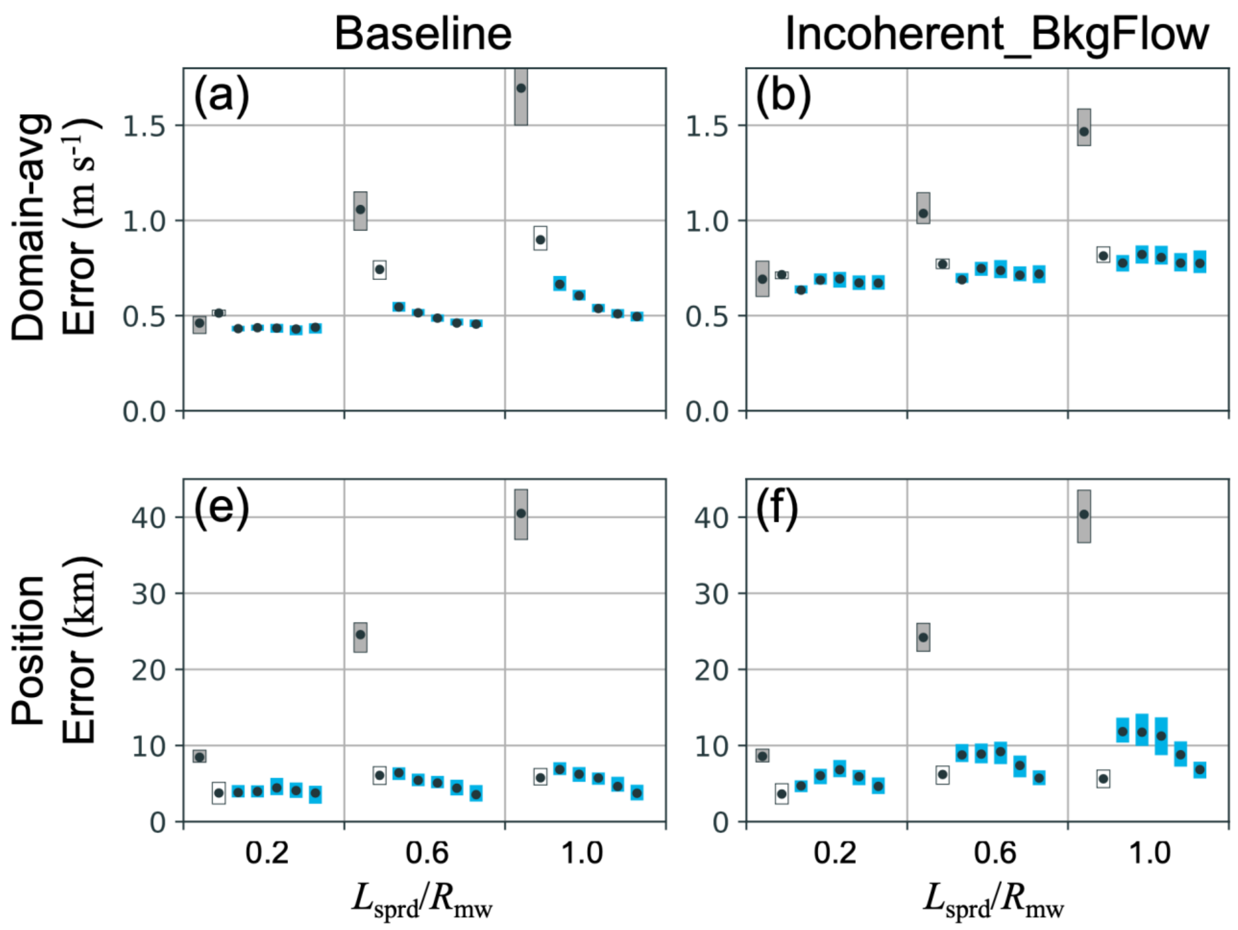
Performance of MSA improves as Ns increases,

Some degradation compared to EnKF is found in linear regimes



Relation between large- and small-scale components

Issue: when background flow errors are incoherent with the vortex position errors (displace to different directions):

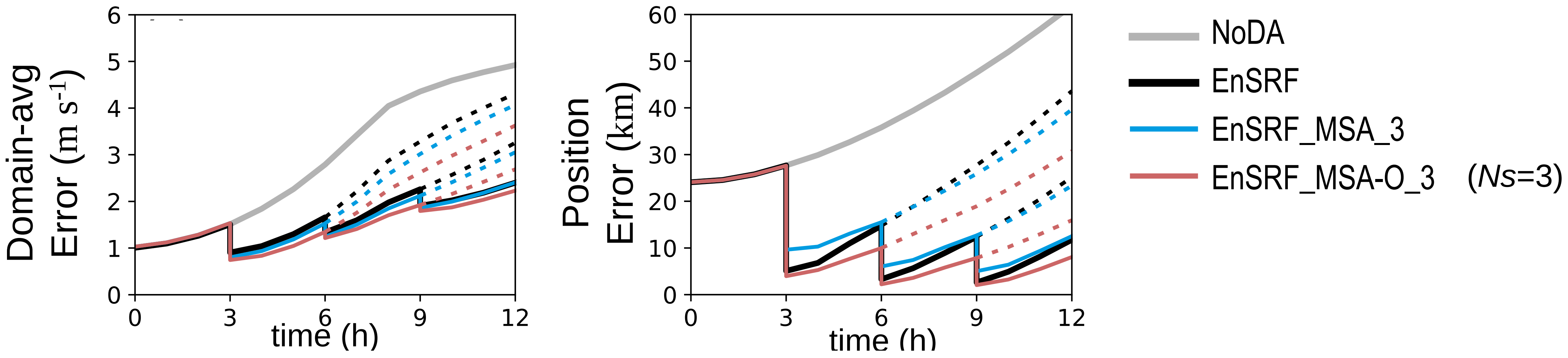


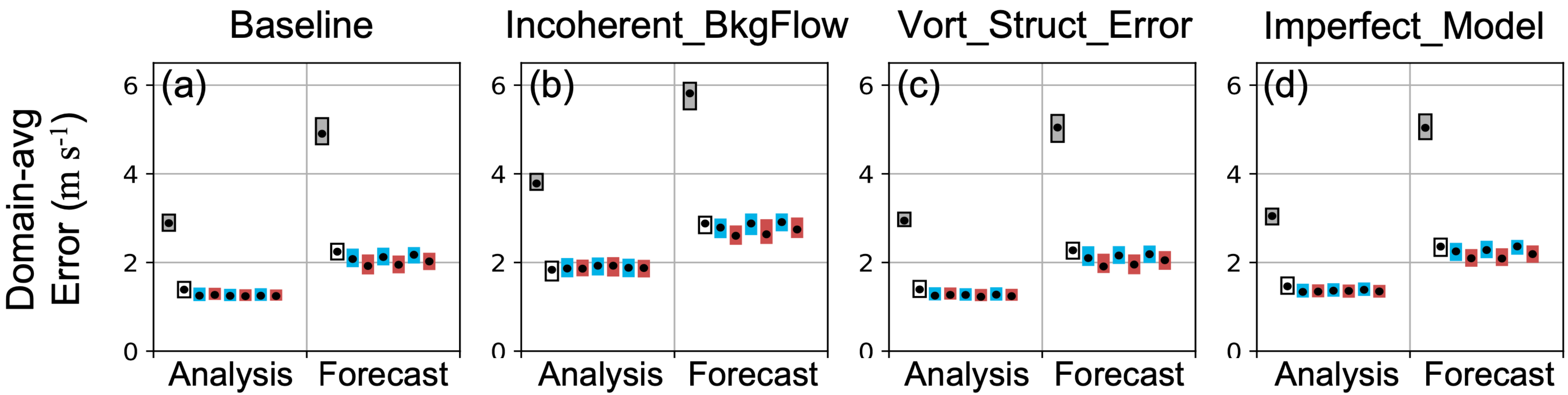
Ying 2019 assumed that small-scale displacement inherits the large-scale ones

$$x_s^b(q_1 + q_2 + \ldots + q_{s-1})$$

Instead, maybe using covariance to update $q_s^b \rightarrow q_s^a$

Performance in a cycling DA experiment





Summary

Motivation and Idea: Position errors in multiscale systems cause a lot of nonlinearity, the multiscale alignment (MSA) approach for ensemble filtering attempts to improve performance.

Stress testing the MSA in a vortex case:
- Improved forecasts as number of scales (*Ns*) increase
- Treatment of irregular lagrangian mesh
- Coherence assumption raises some issue in real applications.

Reference

- Ying, Anderson & Bertino, 2023, DOI: 10.1175/MWR-D-22-0140.1

For more information/discussion: yue.ying@nersc.no